Scottsdale Community College

Basic Arithmetic
Student Workbook

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Second Edition
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A NOTE ABOUT THIS WORKBOOK

This workbook was created through the efforts of three instructors at Scottsdale Community College in Scottsdale, Arizona. Any individual may download and utilize a digital copy of this workbook for free. The Creative Commons licensing of this text allows others to freely use, modify, or remix any of the information presented here. Contact Donna Gaudet at dgaudet.scc@gmail.com if you are an instructor that wishes to teach with these materials or adapt and edit them for teaching.

This workbook does not contain beautiful graphics, hundreds of problems or 4-color printing. Those things are wonderful to have but also very expensive. This workbook does have lessons that were carefully and thoughtfully crafted to lead students on a path to understanding numbers and arithmetic.

As you work through the text, you may find errors. If you do, please contact Donna Gaudet at the email address above and provide specific information about the error and where it is located. The information presented in the text is a work in progress and will evolve with the input of students and other users. We look forward to your feedback!

WORKOOK & SUPPORTING COMPONENTS

This workbook is designed to lead students through a basic understanding of numbers and arithmetic. The included curriculum is broken into twelve lessons (see Table of Contents page for lesson titles). Each lesson includes the following components:

MINILESSON

- The MiniLesson is the main instructional component for each lesson.
- Ideas are introduced with practical situations.
- **Example** problems (marked with a star) are to be completed by watching video links and taking notes/writing down the problem as written by the instructor. Video links can be found at http://sccmath.wordpress.com or may be located within the Online Homework Assessment System.
- **You Try** problems help reinforce lesson concepts and should be worked in the order they appear showing as much work as possible. Answers can be checked in Appendix A.

PRACTICE PROBLEMS

- These problems can be found at the end of each lesson. If you are working through this material on your own, the recommendation is to work all those problems. If you are using this material as part of a formal class, your instructor will provide guidance on which problems to complete. Your instructor will also provide information on accessing answers/solutions for these problems.

ASSESS YOUR LEARNING

- The last part of each lesson is a short assessment. If you are working through this material on your own, use these assessments to test your understanding of the lesson concepts. Take the assessments without the use of the book or your notes and then check your answers. If you are using this material as part of a formal class, your instructor will provide guidance on which problems to complete. Your instructor will also provide information on accessing answers/solutions for these problems.

ONLINE HOMEWORK ASSESSMENT SYSTEM

- If you are using these materials as part of a formal class and your class utilizes an online homework/assessment system, your instructor will provide information as to how to access and use that system in conjunction with this workbook.

TABLE OF CONTENTS

LESSON 1 – WHOLE NUMBERS

INTRODUCTION

We will begin our study of Basic Arithmetic by learning about whole numbers. Whole numbers are the numbers used most often for counting and computation in everyday life.

The table below shows the specific whole-number related objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Identify the *place value* of a digit or digits in a given number.	1, YT7
Read and write *whole numbers*.	2, YT4
Round whole numbers to a given place.	3, YT5, YT6
Rewrite an *exponential* expression in factored form.	8
Compute *numerical expressions* using exponents.	12, 13, YT14
Use correct *order of operations* to evaluate numerical expressions.	9, 10, 11, YT15
Solve whole number *applications* with a problem-solving process	16, YT17

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Whole Numbers
- Number Line
- Place Value
- Round Whole Numbers
- Exponent
- Power
- Factor
- Factored Form
- Mathematical Operations
- Order of Operations (PEMDAS)
- Solution Work Flow
- Complete Solution
- Disjointed Solution
- Problem Solving Process

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

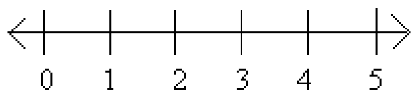
Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

WHOLE NUMBERS, PLACE VALUE, AND ROUNDING

Whole numbers are often referred to as "the counting numbers plus the number 0". The first few *whole numbers* are written as: 0, 1, 2, 3, 4, 5, 6, 7, …

We can place a representative set of these on a number line as follows:



Note that the arrows on the number line indicate that the numbers continue in both directions. We will learn in Lesson 12 about numbers to the left of 0!

We *read whole numbers* from left to right. To do this correctly, we need to know the *place value* of each digit in the number. The table below illustrates place values through the Billions place. You can use this table to help you properly identify number names.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1,	100	10	1,	100	10	1,	100	10	1.

Example 1: Place each number in the chart above. What place value does the digit "0" occupy in each number?

a. 25,032

b. 105,243

c. 12, 340,412

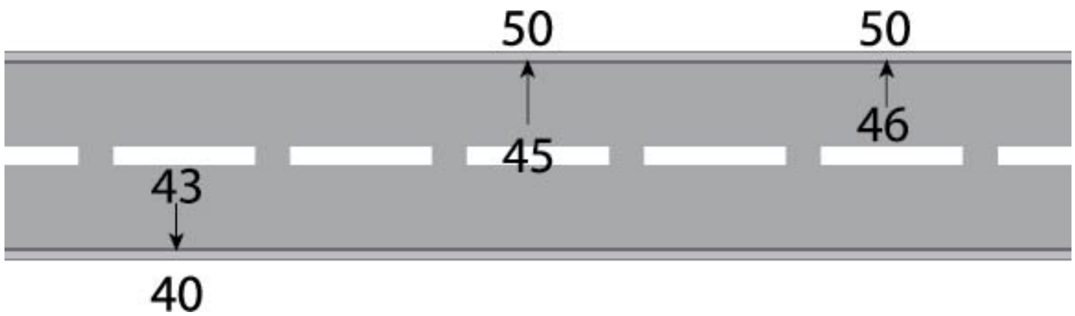
Example 2: Write each whole number below in words.

a. 275,345,201

b. 502,013

To *round* a number means to approximate that number by replacing it with another number that is "close" in value. Rounding is often used when estimating. For example, if I wanted to add 41 and 37, I could round each number to the nearest ten (40 and 40) then add to estimate the sum at 80.

When rounding, the analogy of a road may help you decide which number you are closer to. See the image below. The numbers 43, 45, and 46 are all rounded to the nearest tens place. Note that a number in the middle of the "road" is rounded up.



Example 3:

a. Round 40,963 to the nearest tens place.

b. Round 40,963 to the nearest hundreds place.

c. Round 40,963 to the nearest thousand

d. Round 40,963 to the nearest ten thousand

YOU TRY

4. Write the number 12,304,652 using words.

   ____

5. Round 12,304,652 to the nearest million. ______________________________

6. Round 12,304,652 to the nearest hundred. ______________________________

7. What place does the digit 3 occupy in the number 12,304,652? ________________

EXPONENTS

Exponents are also called *powers* and indicate repeated multiplication.

Worked Example 8: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 3 \cdot 3 = 27 \cdot 3 = 81$

Note: There are 4 *factors* of 3 in the exponential expression 3^4. When we write $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$, we have written 3^4 in *factored form*.

On your calculator, you can compute exponents a couple of ways as follows:

a) If you are raising a number to the second power (for example 4^2), look for an x^2 key on your calculator. Then, enter $4x^2$= or ENTER and you should get 16.

b) If you are raising a number to a power other than 2, look for a carrot key (^). For example 4^5= 4^5= and you should get 1024. Note that you can also use the (^) key even when raising to the 2nd power (also called "squaring").

ORDER of OPERATIONS

Addition, subtraction, multiplication, and division are called *mathematical operations*. When presented with more than one of these in an expression, we need to know which one to address first. The chart below will help us.

P	Simplify items inside Parenthesis ( ), brackets [ ] or other grouping symbols first.
E	Simplify items that are raised to powers (Exponents)
M	Perform Multiplication and Division next
D	(as they appear from **Left to Right**)
A	Perform Addition and Subtraction on what is left.
S	(as they appear from **Left to Right)**

Example 9: Evaluate $8 + 5 \cdot 2$

Example 10: Evaluate $24 \div (4 + 2)$

Example 11: Evaluate $20 - (8 - 2) \div 3 \cdot 4$

Example 12: Evaluate $10 \cdot 3^2 + \dfrac{10-4}{2}$

Example 13: Evaluate $\left(\dfrac{8+2}{7-2}\right)^2$

WORK FLOW AND WRITING SOLUTIONS

When you begin to do work that requires more than a single computation, the steps that you present in your solution should be equivalent. Try to provide a complete solution as seen in the videos as opposed to a series of disjointed computations. See the examples below.

COMPLETE SOLUTION – shows all work in neat, coherent, equivalent steps

$$\begin{aligned} \overset{a}{3+5}-4-2 &= \overset{b}{8-4}-2 \\ &= \overset{c}{4-2} \\ &= \overset{d}{\boxed{2}} \end{aligned}$$

(a) Add 3+5=8.
(b) Subtract 8-4.
(c) Subtract 4-2.
(d) Final answer

DISJOINTED SOLUTION – shows work but steps are not equivalent, information is left out of the solution process

No!

$$\begin{aligned} \overset{a}{3+5}-4-2 &= \overset{b}{8-4} \\ &= 4-2 \\ &= 2 \end{aligned}$$

(a) Add 3+5=8
(b) statement is not true! what happened to -2?
(c) Subtract 4-2
(d) Final answer

MATHEMATICS AND WRITING

When faced with a mathematical problem, you really have two goals. The first is to work the problem correctly and the second is to present a complete solution that can be read and understood by yourself and by others. Just because you know how to do a problem today does not mean that you will quickly remember how to do it when you look back on it in the future. Strive to present complete solutions following the examples and presentations that you see in the media links. Mathematics is really learned through writing. The better your solutions the more you will learn and retain. As you move forward in mathematics, learning to write a good solution may help you solve problems you would not have been able to otherwise.

A NOTE ON NOTATION

In mathematics, the operation of multiplication can be communicated a number of different ways. Each of the notations below means “3 times 5”:

$$3 \text{ x } 5 = (3)(5) = 3(5) = 3\cdot 5$$

YOU TRY

14. Evaluate by hand, showing all possible steps. Try to use good solution flow as discussed on the previous page.

$2+4\cdot 8-(2+3)^2$

Insert check mark to verify same result via calculator: __________

15. Evaluate by hand, showing all possible steps. Try to use good solution flow as discussed on the previous page.

$$\frac{9+3\cdot 7}{5\cdot 2}$$

Insert check mark to verify same result via calculator: __________

APPLICATIONS WITH WHOLE NUMBERS

"Applications" ask you to use math to solve real-world problems. To solve these problems effectively, begin by identifying the information provided in the problem (GIVEN) and determine what end result you are looking for (GOAL). The GIVEN should help you write mathematics that will lead you to your GOAL. Once you have a result, CHECK that result for accuracy then present your final answer in a COMPLETE SENTENCE

Even if the math seems easy to you in this application, practice writing all the steps, as the process will help you with more difficult problems.

Example 16: Amy drives to Costco to buy supplies for an upcoming event. She is responsible for providing breakfast to a large group of Boy Scouts the next weekend. Hashed browns are on her list of supplies to purchase and she needs to buy enough to serve 100 people. The hashed browns are sold in packs of 8 boxes and each box in the pack will serve 4 people. A) How many packs should she buy minimum and B) How many people will she be able to serve with this purchase?

GIVEN: [Write down the information that is provided in the problem. Diagrams can be helpful as well.]

GOAL: [Write down what it is you are asked to find. This helps focus your efforts.]

MATH WORK: [Show your math work to set up and solve the problem.]

CHECK: [Is your answer reasonable? Does it seem to fit the problem? A check may not always be appropriate mathematically but you should always look to see if your result makes sense in terms of the goal.]

FINAL RESULT AS A COMPLETE SENTENCE: [Address the GOAL using a complete sentence.]

YOU TRY

17. You join a local center in your community that has a swimming pool and a group that swims laps each week. The initial enrollment fee is $105 and the group membership is $44 a month. What are your dues for the first year of membership?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

LESSON 1 - PRACTICE PROBLEMS

1. Identify the place value of the digit “6” in each of the following numbers.

    a. 356,230

    b. 6,456,678

    c. 300,560

    d. 461,345,567

    e. 6,540,345,234

    f. 405,978,106

2. Write each of the following whole numbers in words.

    a. 356,230

    b. 6,456,678

    c. 300,560

    d. 461,345,567

    e. 6,540,345,234

    f. 405,978,106

3. Write each of the following in “factored” form and then compute the final result.

    a. 2^3

    b. 3^5

    c. 4^2

    d. 5^4

    e. 6^3

    f. 7^2

4. Evaluate each of the following using correct order of operations. Show all possible steps and check your work using your calculator.

   a. $7 + 2(5 - 3)$

   b. $4 + 12 \div 3 + 2 \bullet 4^2$

   c. $3^2 + 6 - 5$

   d. $51 - (47 - 2) \div 5 \bullet 3$

   e. $15 + [3(8 - 2^2) - 6]$

   f. $4 + 2 \times 6^3 + \dfrac{8-2}{2+1}$

5. Solve each of the following applications showing as much work as possible. Use the problem-solving process described in the lesson to write your solution.

   a. Mark deposited \$450, \$312, \$125, and \$432 in his bank account this month. He also made deductions of \$205 and \$123. If his balance at the beginning of the month was \$1233, what was his balance at the end of the month?

b. Jenelle financed a 2012 Chevy Camaro on 60-month terms for $673 per month. If the MSRP on the car was $35,000 and she put no money down, how much over the MSRP did she end up paying?

c. In the winter, the farmer's market sees an average of 1516 visitors each Sunday. In the summer, they see an average of 4278 visitors each Sunday. How many more visits are there in the summer than in the winter (on average)?

d. There are 12 reams of paper in a given box. How many reams are there in 25 boxes?

LESSON 1 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can.

1. Identify the place value of the digit "7" in the following number.

11,261,971

2. Identify the place value of the digit "4" in the following number.

5,241,966

3. Write the following number in words.

12,089,526

4. Write the following number in words.

8,002,079,123

5. Write the number in "factored" form and then compute the final result

4^2

6. Evaluate the following using correct order of operations. Show all possible steps and check your work using your calculator.

$$9 \div (7-4) + 3^2 \cdot 6 - 5$$

7. Evaluate the following using correct order of operations. Show all possible steps and check your work using your calculator.

$$12 \div 4 \cdot 7 - \frac{14-4}{7-2} + 3 \cdot 4$$

8. Solve the following problem using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

Amy deposited \$325, \$473, \$224 and \$653 into her checking account one month. She paid bills in the amounts of \$54, \$127, \$96 and \$685. How much money does she have left over after paying her bills?

9. Solve the following problem using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

Tally bought dog food for an animal rescue shelter. She bought 6 bags that weighed 25 pounds each and 19 bags that weighed 7 pounds each. How many pounds of dog food did she buy?

LESSON 2 – INTRODUCTION TO FRACTIONS

INTRODUCTION

In this lesson, we will work again with factors and also introduce the concepts of prime and composite numbers. We will then begin working with fractions and the concept of "parts of a whole".

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Find all *factors* of a number.	1, YT4
Write the *prime factorization* for a *composite number*.	2, YT5
Find the *LCM* (Least Common Multiple) of two numbers.	3
Recognize a fraction as part of a whole.	6
Identify fractions represented by shading.	6
Write *mixed numbers* as *improper fractions*.	9, YT10, YT14
Write *improper fractions* as *mixed numbers*.	11, YT12, YT13
Find *equivalent fractions*.	15, 16, YT18
Write fractions in simplest form.	17, YT 18, YT19
Compare fractions.	20
Graph fractions on a number line.	21
Simplify fractions that have a "1" in the numerator or denominator.	22
Simplify fractions that have a "0" in the numerator or denominator.	22
Solve *applications* with fractions.	23, YT24

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Factors
- Prime Number
- Composite Number
- Prime Factorization
- Exponential Form
- Factored Form
- Least Common Multiple (LCM)
- Fraction
- Numerator
- Denominator
- Proper Fraction
- Improper Fraction
- Mixed Number
- Quotient
- Remainder

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

FACTORS

The *factors* of a number divide the number evenly (with remainder zero).

Example 1: Find all factors of 24.

PRIME FACTORIZATION

A *prime number* is a whole number that has only itself and 1 as factors.

(Example: 2, 3, 5, 7, 13, 29, etc…)

A *composite number* is a whole number that is not prime (i.e. has factors other than itself and 1). Every composite number can be written as a product of prime factors. This product is called the *prime factorization*.

Example 2: Find the prime factorization of each of the following. Write the final result in *exponential form* and *factored form.*

72 600

LEAST COMMON MULTIPLE (LCM)

Example 3: The *LCM* of two numbers is the smallest number for which both numbers are factors. For example, the LCM of 2 and 4 is 4. The LCM of 3 and 5 is 15. Find the LCM of 8 and 10:

Multiples of 8 are: ____

Multiples of 10 are: ____

Some common multiples of 8 and 10 are: __________________________________

The LEAST COMMON MULTIPLE of 8 and 10 is _______________

YOU TRY

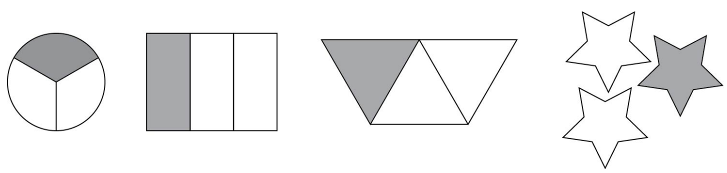
4. List the factors of 18.

5. Find the prime factorization of 270.

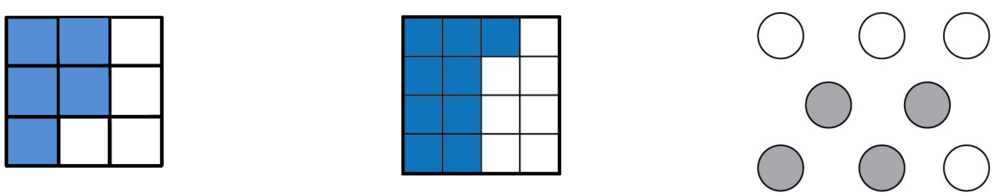
FRACTIONS

Suppose I buy a candy bar to split with two of my friends. What number could we use to discuss how much of the bar each of us would get? Well, if we have 1 bar and it is split into 3 equal pieces, then we would say that each person gets $\frac{1}{3}$ of the bar. The number $\frac{1}{3}$ is called a *fraction* because we use it to represent part (one part) of a whole (3 pieces).

The fraction $\frac{1}{3}$ can be represented by the shaded part in each of the following diagrams. Notice that in each diagram, the whole is a different shape or set of shapes but the use of the fraction $\frac{1}{3}$ still applies.



Example 6: Identify the fraction represented by the shaded part of each figure.



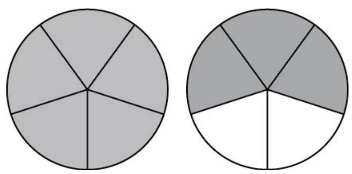
YOU TRY

7. Draw two different figures or sets of figures that are $\frac{3}{4}$ shaded.

Vocabulary of *fractions*:

- The top number in a fraction is called the *numerator.*
- The bottom number in a fraction is called the *denominator.*
- Fractions for which the top number is smaller than the bottom are called *proper fractions.*
- Fractions whose numerator is larger than the denominator are called *improper fractions* and can be written as what are called *mixed numbers*.

Example 8: Identify the fraction represented by the shaded part of each figure.



Example 9: Express as an improper fraction.

$2\frac{1}{4}$ $12\frac{1}{3}$

YOU TRY

10. Write the steps to convert a *mixed number* to an *improper fraction* (from video above)

Example 11: Express as a mixed number.

a. $\frac{42}{5}$ b. $\frac{53}{9}$ c. $\frac{84}{7}$

YOU TRY

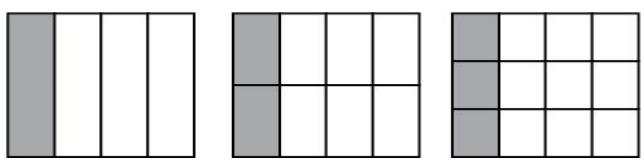
12. Write the steps to convert an *improper fraction* to a *mixed number* (from video above)

13. Express $\frac{57}{11}$ as a mixed number.

14. Express $8\frac{1}{5}$ as an improper fraction.

EQUIVALENT FRACTIONS

Each rectangle below has the same amount of shaded area. The simplest way to represent the shaded areas as a fraction is as $\frac{1}{4}$. All of the listed fractions are equivalent to $\frac{1}{4}$.



$\frac{1}{4}$ $\frac{2}{8}$ $\frac{3}{12}$

Example 15: Which of the given fractions are equivalent to $\frac{2}{7}$?

$\frac{4}{14}$ $\frac{6}{18}$ $\frac{10}{35}$ $\frac{14}{28}$

Example 16: Find four fractions equivalent to $\frac{1}{5}$.

FRACTIONS IN SIMPLEST FORM

Fractions are in simplest form if they are completely reduced. To completely reduce a fraction, remove all common factors other than 1 from the numerator and denominator. Leave fraction answers always in simplest form.

Example 17: Write the following fractions in simplest form.

$\frac{4}{16}$ $\frac{28}{54}$ $\frac{360}{495}$

YOU TRY

18. Find two fractions equivalent to $\frac{3}{8}$.

19. Write $\frac{40}{72}$ in simplest form.

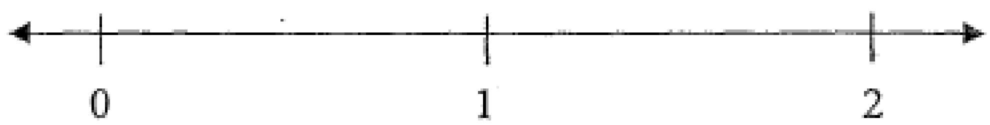
COMPARING FRACTIONS

To compare fractions, create equivalent fractions with the same denominator then compare the numerators.

Example 20: Which is larger, $\frac{4}{5}$ or $\frac{6}{7}$?

GRAPHING FRACTIONS ON A NUMBER LINE

Example 21: Divide the given line into units of length $\frac{1}{6}$, label each tick mark, then plot and label the following numbers: $\frac{0}{6}, \frac{2}{6}, \frac{5}{6}, \frac{6}{6}, \frac{9}{6}$. Provide alternate forms if possible.



ONE and ZERO

Example 22: Complete the following table.

Number	Computation	Simplified Result	General Rule
$\frac{4}{1}$			
$\frac{4}{4}$			
$\frac{0}{4}$			
$\frac{4}{0}$			

APPLICATION OF FRACTIONS

Example 23: There are 14 men and 12 women in Professor Bohart's MAT082 class. What fraction of the students in the class are women?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

YOU TRY

24. The local PTA group approved a fall carnival by a vote of 15 to 5. What fraction of the PTA group voted against the bill? Remember to reduce the final result.

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

LESSON 2 - PRACTICE PROBLEMS

1. List all the factors for each number.

a. 12 b. 23 c. 62 d. 81

e. 144 f. 28 g. 71

2. For each of the numbers in problem 1 above, indicate whether it is composite or prime and why. For each composite, write its prime factorization.

a. 12 b. 23 c. 62 d. 81

e. 144 f. 28 g. 71

3. Draw accurate diagrams that represent each of the fractions below.

a. $\frac{5}{8}$

b. $2\frac{1}{3}$

c. $\frac{6}{5}$

d. $1\frac{1}{5}$

e. $\frac{0}{2}$

4. For each problem below, write the fraction that best describes the situation. Be sure to reduce your final result.

a. John had 12 marbles in his collection. Three of the marbles were Comet marbles. What fraction of the marbles were Comet marbles? What fraction were NOT Comet marbles?

b. Jorge's family has visited 38 of the 50 states in America. What fraction of the states have they visited?

c. In a given bag of M & M's, 14 were yellow, 12 were green, and 20 were brown. What fraction were yellow? Green? Brown?

d. Donna is going to swim 28 laps. She has completed 8 laps. What fraction of laps has she completed? What fraction of her swim remains?

e. Last night you ordered a pizza to eat while watching the football game. The pizza had 12 pieces of which you ate 6. Today, two of your friends come over to help you finish the pizza and watch another game. What is the fraction of the LEFTOVER pizza that each of you gets to eat (assuming equally divided). What is the fraction of the ORIGINAL pizza that each of you gets to eat (also assuming equally divided).

5. Write each mixed number as an improper fraction.

a. $3\frac{1}{4}$ b. $2\frac{1}{3}$ c. $6\frac{4}{5}$

d. $1\frac{1}{7}$ e. $7\frac{1}{2}$

6. Write each improper fraction as a mixed number.

a. $\frac{16}{13}$ b. $\frac{17}{3}$ c. $\frac{42}{25}$

d. $\frac{73}{7}$ e. $\frac{21}{2}$

7. Which of the following CANNOT be written as a mixed number and why?

a. $\frac{8}{3}$ b. $\frac{15}{8}$ c. $\frac{21}{25}$

d. $\frac{34}{27}$ e. $\frac{11}{12}$

8. Write two equivalent fractions for each of the fractions below.

a. $\frac{3}{7}$ b. $\frac{4}{5}$ c. $\frac{2}{9}$

d. $\frac{5}{8}$ e. $\frac{11}{12}$

9. Write each fraction in simplest form.

a. $\frac{3}{6}$ b. $\frac{15}{5}$ c. $\frac{12}{36}$

d. $\frac{120}{164}$ e. $\frac{11}{11}$ f. $\frac{0}{21}$

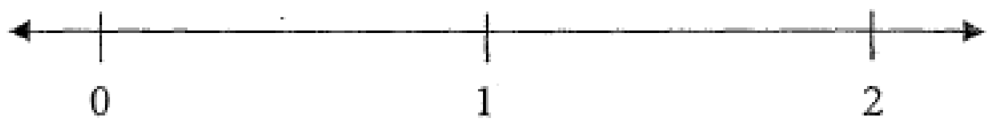
10. Find the LCM of each of the pairs of numbers below.

a. 4 and 5 b. 6 and 12 c. 5 and 7

d. 3 and 8 e. 12 and 15

11. Using equally spaced tick marks, plot the following numbers on the number line.

$$\frac{1}{8} \quad \frac{8}{8} \quad \frac{11}{8} \quad \frac{0}{8} \quad \frac{3}{4}$$



12. For each pair of fractions, place < or > or = between them to show the relationship between the two numbers.

a. $\frac{3}{7} \quad \frac{1}{3}$ b. $\frac{3}{5} \quad \frac{1}{2}$ c. $\frac{11}{13} \quad \frac{6}{7}$

d. $\frac{3}{4} \quad \frac{6}{8}$ e. $\frac{5}{9} \quad \frac{2}{3}$

13. Simplify each of the following fractions if possible.

a. $\frac{5}{1}$ b. $\frac{6}{6}$ c. $\frac{0}{4}$

d. $\frac{1}{6}$ e. $\frac{1}{1}$ f. $\frac{1}{0}$

LESSON 2 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can.

1. List all the factors of 24.

2. List all the factors of 60.

3. Is 51 composite or prime? Explain.

4. Is 73 composite or prime? Explain.

5. Draw an accurate diagram that represents. $\frac{4}{9}$

6. Draw an accurate diagram that represents. $\frac{9}{4}$

7. Raymond has 23 homework problems and has completed 17 problems. What fraction of problems has he completed? What fraction of problems does he have left to complete?

8. Bill bought 2 cakes for a party. Each cake had 12 slices. If 15 slices of cake were eaten, what fraction of the **two cakes** was eaten? What fraction of **one cake** was eaten?

9. Write the mixed number $3\frac{3}{5}$ as an improper fraction.

10. Write $\frac{17}{12}$ as a mixed number.

11. Write two fractions that are equivalent to $\frac{5}{9}$.

12. Write the fraction $\frac{18}{30}$ in simplest form.

13. Find the LCM of 9 and 15.

14. Place < or > or = between the two numbers to show the correct relationship.

$$\frac{7}{9} \qquad \frac{7}{11}$$

LESSON 3 – FRACTION ADDITION & SUBTRACTION

INTRODUCTION

In this lesson, we will concentrate on combining fractions through addition and subtraction (and save multiplication and division for the NEXT lesson!). When combining fractions through addition or subtraction, the idea of "unit fractions" and "copies of unit fractions" takes center stage. Look for this phrasing as you work through the MiniLesson.

The table below shows the specific whole-number related objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Examples
Add and subtract fractions with like denominators.	1, 2, YT3
Add and subtract fractions with unlike denominators.	4, 5, YT6, 10, 11
Add and subtract mixed numbers.	7, YT8, 12, YT13
Explain the difference between CD, LCM, & LCD	WE9
Solve applications using fraction addition and subtraction.	14, YT15
Use correct order of operations when adding/subtracting fractions	16, YT17

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Fraction
- Unit Fraction
- Common Denominator
- Least Common Denominator (LCD)
- Least Common Multiple (LCM)
- Mathematical operations
- PEMDAS

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

ADDING/SUBTRACTING FRACTIONS WITH LIKE DENOMINATORS

Suppose Josh ordered a pizza and sat down to watch football. The pizza was cut into 6 equal slices. During the first half of the game, he ate one slice and during the second half, he ate another. How much of the pizza did Josh eat?

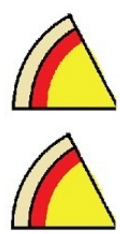
2 slices out of 6 or $\frac{2}{6}=\frac{1}{3}$ of the pizza. (Remember to fully reduce!)

We can also look at it another way. Since the pizza was divided into 6 pieces, the "units" of the pizza division were sixths. The fraction $\frac{1}{6}$ is a *unit fraction*. How many copies of the $\frac{1}{6}$ size pieces did he eat? He ate 2 separate $\frac{1}{6}$-size pieces so he ate:

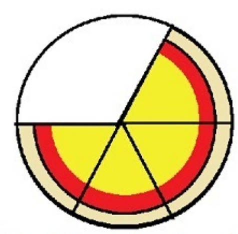
$$\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3} \text{ of the pizza.}$$

1 whole pizza

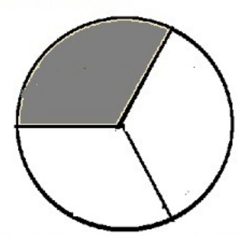
1 whole pizza equals 6 copies of 1/6 of a pizza



Josh ate 2 copies of 1/6 of a pizza or 2/6 of a pizza



The white portion shows how much Josh ate



The shaded portion is equivalent to what Josh ate 2/6 of the pizza equals 1/3 of the pizza

Let's see how the example above helps us with the problems below.

Example 1: Add. Write your answer in simplest form.

a. $\frac{5}{12}+\frac{2}{12}=$

b. $\frac{3}{8}+\frac{7}{8}=$

Example 2: Subtract. Write your answer in simplest form.

a. $\frac{5}{8}-\frac{1}{8}=$

b. $\frac{11}{12}-\frac{7}{12}=$

YOU TRY

3. Perform the indicated operations. Write your answer in simplest form.

a. $\frac{4}{9}+\frac{2}{9}=$

b. $\frac{12}{13}-\frac{6}{13}$

ADDING/SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

Let's look at another situation. Suppose Josh had two trays of brownies to serve for desert at an upcoming party. One tray had chocolate brownies and the other had butterscotch.

The chocolate tray was cut into 12 equal pieces and the butterscotch tray into 6 equal pieces (big brownies!). Josh ate one brownie from each tray before his guests arrived. What fraction of the brownies on each tray did Josh eat? Josh ate:

$\frac{1}{12}$ of the chocolate brownie tray $\qquad$ $\frac{1}{6}$ of the butterscotch brownie tray

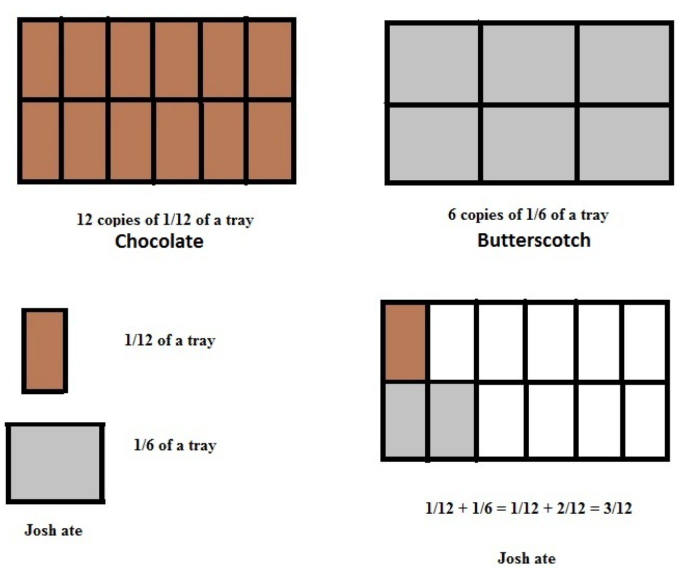
What fraction of the brownies on a single tray did Josh eat? Josh ate:

$$\frac{1}{12}+\frac{1}{6} \text{ of the brownies on a single tray}$$

Hmmm…how do we combine those? Did Josh eat $\frac{2}{18}=\frac{1}{9}$ of the brownies? No, that does not make sense. What if we divide the butterscotch tray into 12 pieces just like the chocolate tray? Josh originally had $\frac{1}{6}$ of the butterscotch tray, which would be the same as $\frac{2}{12}$ of the tray. Now can we combine the amounts?

$$\frac{1}{12}+\frac{2}{12}=\frac{3}{12}$$

Yes! Now that makes more sense. Josh ate $\frac{1}{12}$ of the brownies on one tray and $\frac{2}{12}$ of the brownies on the other tray for a total of $\frac{3}{12}$ of the brownies on a single tray.



What we learned from the brownie example on the previous page is that to add two fractions they must have the same (common) denominator. Let's see how this works with some additional examples.

Example 4: Add. Start by identifying a common denominator (CD). Write your answer as both an improper fraction and a mixed number if possible.

a. $\frac{1}{2}+\frac{2}{3}=$

b. $\frac{3}{8}+\frac{5}{6}=$

Example 5: Subtract. Start by identifying a common denominator (CD). Write your answer as both an improper fraction and a mixed number if possible.

a. $\frac{2}{3}-\frac{1}{2}=$

b. $\frac{3}{4}-\frac{2}{5}=$

YOU TRY

6. Perform the indicated operations. Start by identifying a common denominator (CD). Write your answer as both an improper fraction and a mixed number if possible.

a. $\frac{4}{5}+\frac{3}{8}=$

b. $\frac{4}{5}-\frac{11}{15}=$

Rules for adding or subtracting fractions.

1. To add or subtract fractions with LIKE denominators, add or subtract the numerators as indicated and place over the like denominator. Reduce if possible.
2. To add or subtract fractions with UNLIKE denominators, obtain a common denominator then follow step 1 above. Be sure to reduce if possible.

Let's see how these rules work when we start using mixed numbers.

ADDING AND SUBTRACTING MIXED NUMBERS

Example 7: Perform the indicated operation. Start by identifying a common denominator (CD). Write your answer as both an improper fraction and a mixed number if possible.

a. $1\frac{1}{5}+2\frac{2}{3}=$

b. $4\frac{3}{5}-1\frac{5}{6}=$

YOU TRY

8. Perform the indicated operation. Start by identifying a common denominator (CD). Write your answer as both an improper fraction and a mixed number if possible.

a. $4\frac{1}{3}-1\frac{3}{4}=$

b. $6\frac{1}{2}+3\frac{5}{8}=$

LCM vs. LCD vs. CD

Worked Example 9: Let's distinguish between three vocabulary terms related to fractions.

- LCD = Least Common Denominator
- LCM = Least Common Multiple
- CD = Common Denominator

Problem: Add $\frac{1}{2}+\frac{1}{8}$	To start this problem, what we need is a *Common Denominator (CD)* of the denominators 2 and 8. What number works? Well, we need a number that is divisible by both 2 and 8. Let's try 2 times 8, which is 16. That works and guides the solution below:
Step 1 $\frac{1}{2}+\frac{1}{8}$ Step 2 $=\frac{1\cdot 8}{2\cdot 8}+\frac{1\cdot 2}{8\cdot 2}$ Step 3 $=\frac{8}{16}+\frac{2}{16}$ Step 4 $=\frac{10}{16}=\frac{5}{8}$	Step 1: Original problem Step 2: Determine the multiplier to convert each fraction to the common denominator Step 3: Convert each fraction to the common denominator Step 4: Combine the numerators over the common denominator then reduce to the final result.

However, notice that we had to reduce at the end. Let's try another track. Let's first find the LCM or Least Common Multiple of the denominators 2 and 8. Remember how that was done previously? List multiples of 2 and 8 to find the first one in common:

2: 2, 4, 6, **8**, 10, 12, 14, 16… and 8: **8**, 16, 24, 32,….

We can see that 8 is the LCM of 2 and 8. It is the smallest number that both 2 and 8 divide into evenly. Thus, it also fits the definition of the *LCD* or *Least Common Denominator*. LCD and LCM are really equal in a sense. Let's see what happens when we use the LCD:

Step 1 $\frac{1}{2}+\frac{1}{8}$ Step 2 $=\frac{1\cdot 4}{2\cdot 4}+\frac{1}{8}$ Step 3 $=\frac{4}{8}+\frac{1}{8}=\frac{5}{8}$	We get the same result as above AND in fewer steps with no reducing required at the end. So, the motto is, use the LCD whenever possible and remember that the LCD and the LCM are the same number.

SPECIAL CASES/CONFUSING SITUATIONS

Each of the examples below displays a little "oddity" or unusual situation that you may not be sure how to deal with the first time you see it. Refer back to these examples as needed while completing the HW or Practice problems.

Example 10: What if your numerator ends up being 0?

$$\frac{3}{5}-\frac{9}{15}$$

Example 11: What if your numerator ends up the same as your denominator?

$$\frac{1}{4}+\frac{6}{8}$$

Example 12: What if your numerator is a multiple of your denominator?

$$2\frac{1}{3}+3\frac{4}{6}$$

YOU TRY

13. Perform the indicated operation. Write your answer in simplest form.

a. $4\frac{2}{3}-3\frac{4}{6}$

b. $4\frac{1}{3}+2\frac{2}{3}$

APPLICATIONS OF FRACTION ADDITION

Example 14: In a bag of 100 M & M's, 30 are brown, 20 are yellow, 20 are red, 10 are green, 10 are orange, and 10 are blue. What fraction of the M & M's are brown, or blue?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

YOU TRY

15. On a recent trip, Robert flew $4\frac{2}{3}$ hours on his first flight and $12\frac{1}{2}$ hours on his second flight. How many hours was he in the air? Leave your final answer as a mixed number.

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

ORDER OF OPERATIONS WITH FRACTIONS

Remember our order of operations from Lesson 1? We will use the same order when working with fraction expressions that involve multiple operations.

P	Simplify items inside Parenthesis ( ), brackets [ ] or other grouping symbols first.
E	Simplify items that are raised to powers (Exponents)
M	Perform Multiplication and Division next
D	(as they appear from **Left to Right**)
A	Perform Addition and Subtraction on what is left.
S	(as they appear from **Left to Right)**

Example 16: Use order of operations to simplify the expression showing all possible steps.

a. $\frac{5}{8}+\frac{3}{4}-\frac{4}{5}=$

b. $\left(3-\frac{1}{4}\right)-\left(\frac{1}{3}+\frac{1}{15}\right)=$

YOU TRY

17. Perform the indicated operations. Write your answer in simplest form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

$\left(\frac{1}{5}+\frac{1}{15}\right)+\left(\frac{7}{8}-\frac{1}{4}\right)=$

LESSON 3 - PRACTICE PROBLEMS

1. Add or subtract each of the following. Be sure to leave your answer in simplest (reduced) form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{5}{8}+\frac{4}{8}$

b. $\frac{4}{3}-\frac{1}{3}$

c. $\frac{2}{10}+\frac{3}{10}$

d. $\frac{7}{22}+\frac{5}{22}$

e. $\frac{12}{17}-\frac{3}{17}$

2. Add or subtract each of the following. State clearly what the common denominator is. Be sure to leave your answer in simplest (reduced) form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{5}{7}+\frac{4}{9}$

b. $\frac{4}{5}-\frac{1}{3}$

c. $\frac{2}{3}+\frac{3}{5}$

d. $\frac{7}{12}+\frac{5}{24}$

e. $\frac{4}{5}-\frac{3}{7}$

3. Perform the indicated operation. Write your answer in simplest form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $1\frac{3}{7}+2\frac{3}{8}$

b. $2\frac{4}{5}-1\frac{1}{3}$

c. $3\frac{2}{3}+1\frac{3}{5}$

d. $2\frac{7}{12}+3\frac{5}{24}$

e. $4\frac{4}{5}-2\frac{3}{7}$

4. Perform the indicated operations. Write your answer in simplest form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{1}{2}-\frac{1}{3}+\frac{1}{4}$

b. $2-\frac{8}{5}$

c. $\frac{2}{3}+\frac{1}{3}-\frac{1}{4}$

d. $\left(2-\frac{1}{3}\right)+\left(\frac{2}{3}+\frac{1}{15}\right)$

5. Solve each of the following application problems using the 5-step process illustrated in the lesson. Leave final answers in mixed number form if possible.

a. If Josh ate $\frac{1}{4}$ of a pizza, what fraction of the pizza is left?

b. If I drove $10\frac{2}{3}$ miles one day and $12\frac{1}{4}$ miles the second day and $8\frac{1}{5}$ miles the third day, how far did I drive?

c. Melody bought a 2-liter bottle of soda at the store. If she drank $\frac{1}{8}$ of the bottle and her brother drank $\frac{2}{7}$ of the bottle, how much of the bottle is left?

d. James brought a small bag of carrots for lunch. There are 6 carrots in the bag. Is it possible for him to eat $\frac{2}{6}$ of the bag for a morning snack and $\frac{5}{6}$ of the bag at lunch? Why or why not?

e. Suppose that David is able to tile $\frac{1}{4}$ of his floor in 3 hours. How long would it take him to tile the rest of the floor? Use addition to solve this and not multiplication.

6. On the left side, EXPLAIN the mistake made in the problem. On the right side, WORK the problem correctly.

Explain the mistake made in the problem below:	Work the problem correctly:
$\frac{2}{3}+\frac{1}{5}=\frac{3}{8}$	$\frac{2}{3}+\frac{1}{5}=$

LESSON 3 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Add the fractions. Be sure to leave your answer in simplest (reduced) form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{4}{9}+\frac{2}{9}$

b. $\frac{12}{7}+\frac{19}{7}$

c. $\frac{1}{2}+\frac{2}{3}$

d. $\frac{7}{8}+\frac{5}{12}$

2. Subtract the fractions. Be sure to leave your answer in simplest (reduced) form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{8}{11}-\frac{5}{11}$

b. $\frac{27}{4}-\frac{17}{4}$

c. $\frac{2}{3}-\frac{2}{5}$

d. $\frac{25}{4}-\frac{9}{8}$

3. Add the mixed numbers. Write your answer in simplest form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $5\frac{2}{7}+3\frac{5}{7}$

b. $2\frac{1}{6}+4\frac{1}{3}$

4. Subtract the mixed numbers. Write your answer in simplest form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $27\frac{3}{8} - 12\frac{7}{8}$

b. $5\frac{1}{7} - 3\frac{3}{5}$

5. Perform the indicated operations. Write your answer in simplest form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

$$\left(10\frac{2}{3} + 2\frac{1}{5}\right) - \left(4\frac{1}{15} + 3\right)$$

6. Solve the following problem using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

Maureen went on a 3 day, 50 mile biking trip. The first day she biked $21\frac{2}{3}$ miles. The second day she biked $17\frac{3}{8}$ miles. How many miles did she bike on the 3rd day?

7. Solve the following problem using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

Scott bought a 5 lb bag of cookies at the bakery. He ate $\frac{2}{5}$ of a bag and his sister ate $\frac{2}{9}$ of a bag. What fraction of the bag did they eat? What fraction of the bag remains?

LESSON 4 – FRACTION MULTIPLICATION & DIVISION

INTRODUCTION

Now that we have learned how to add and subtract fractions, we will work with the remaining two major mathematical operations: multiplication and division.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Multiply fractions	1, YT2
Solve applications of fraction multiplication	3, YT4, YT14
Divide fractions	5, YT6
Solve applications of fraction division	7, YT8
Use correct *order of operations* when working with fractions	9, 10, YT11
Compute *special cases* with fractions (multiply or divide by 0).	12, 13

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Multiply Fractions
- Of
- Divide Fractions
- Reciprocal
- Exponent
- Evaluate
- Order of Operations
- PEMDAS
- Multiplication by Zero
- Division by Zero

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

MULTIPLYING FRACTIONS

When would we ever need to multiply fractions? Let's go back to the first pizza example in the last lesson. There, we looked at the following example as an addition problem.

Word Description	Mathematical Computations ADDITION
Josh ate slices of size $\frac{1}{6}$ and $\frac{1}{6}$	Josh ate $\frac{1}{6}+\frac{1}{6}=\frac{2}{6}$ or $\frac{1}{3}$ of the pizza

What if we look at this as multiplication instead? How would that look different?

Word Description	Mathematics Computations MULTIPLICATION
Josh ate 2 slices of size $\frac{1}{6}$	Josh ate $2\cdot\frac{1}{6}=\frac{2}{1}\cdot\frac{1}{6}=\frac{2\cdot 1}{1\cdot 6}=\frac{2}{6}=\frac{1}{3}$ of the pizza

Let's break apart the mathematics step-by-step and see what happens:

$2\cdot\frac{1}{6}$	Original problem
$=\frac{2}{1}\cdot\frac{1}{6}$	Write 2 as a fraction
$=\frac{2\cdot 1}{1\cdot 6}$	Multiply straight across
$=\frac{2}{6}$	Compute multiplication
$=\frac{1}{3}$	Reduce

<u>Steps to multiply fractions (short list):</u>
1. Convert any whole numbers to fractions.
2. Multiply straight across and reduce if possible.

Notice that when multiplying, we do NOT obtain a common denominator. Let's apply these rules to some examples and see how more complicated problems work.

Example 1: Multiply each of the following. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{1}{4}\cdot\frac{3}{2}=$

b. $\frac{5}{8}\cdot 4=$

c. $2\frac{1}{5}\cdot 3\frac{1}{9}=$

d. $\frac{6}{12}\cdot\frac{14}{24}=$

e. $\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{3}{4}=$

f. $1\frac{1}{4}\cdot 3\frac{2}{3}\cdot\frac{3}{5}=$

Let's modify our fraction multiplication rules to include the new ideas in the examples.

<u>Steps to multiply fractions (full list):</u>
1. Convert any whole numbers to fractions.
2. Convert any mixed numbers to improper fractions
3. Multiply straight across.
4. Reduce along the way if possible.
5. Present final, reduced answer at the end.

NOTE: We do not obtain a common denominator when multiplying fractions!

YOU TRY

2. Multiply each of the following. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{4}{5}\cdot 10=$

b. $2\frac{1}{2}\cdot 1\frac{3}{4}=$

APPLICATIONS OF FRACTION MULTIPLICATION

Example 3: Matt was training for a marathon and had a 20-mile run listed on his training calendar. If he only completed $\frac{3}{4}$ of the run, how far did he go?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

In mathematics, the word "of" often implies multiplication as shown in the examples above.

YOU TRY

4. Darcy's iPod has 3525 songs on it. If $\frac{1}{3}$ of those songs are categorized as Hip Hop/Rap, how many Hip Hop/Rap songs are on Darcy's iPod?

GIVEN:

GOAL:

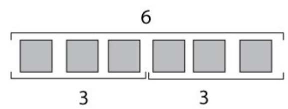
MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

DIVIDING FRACTIONS

Before we discuss division with fractions, let's take a step back and talk just about division in general. What does the operation of division do? Take a look at the problem below.

Problem	Explanation	Result	Check
$6 \div 3$	 There are exactly 2 pieces of size 3 inside 6.	$6 \div 3 = 2$	$2 \cdot 3 = 6$

Are division and multiplication somehow related? Could we turn $6 \div 3$ into a multiplication problem? Let's see how that would be done.

Problem	Multiplication Steps	Result	Check
$6 \div 3$	$6 \div 3 = 6 \cdot \frac{1}{3} = \frac{6}{3} = 2$	$6 \div 3 = 2$	$2 \cdot 3 = 6$

What did we do? We multiplied 6 by the *reciprocal* of 3. That is, we multiplied 6 by $\frac{1}{3}$ and we achieved the same result as $6 \div 3$. Division, then, can be converted to multiplication by using a *reciprocal.*

Let's see if this process makes logical sense when we divide by a fraction.

> Suppose you want to share a candy bar with 3 friends. You know each of your friends would get $\frac{1}{3}$ of the bar. You want to be sure so you ask, "how many pieces of size $\frac{1}{3}$ are there in one candy bar?"

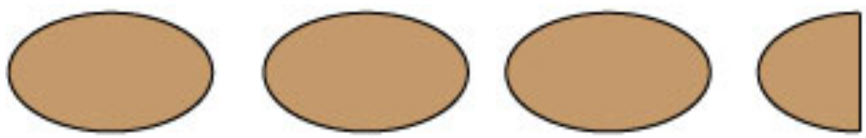
Problem	Multiplication Steps	Result	Check
$1 \div \frac{1}{3}$	$1 \div \frac{1}{3} = 1 \cdot 3 = 3$	$1 \div \frac{1}{3} = 3$	$3 \cdot \frac{1}{3} = \frac{3}{3} = 1$

We changed division by $\frac{1}{3}$ to multiplication by 3 (the reciprocal of $\frac{1}{3}$) giving us a result of 3. There are indeed 3 pieces of size $\frac{1}{3}$ inside one full candy bar. Our division process works for fractions as well.

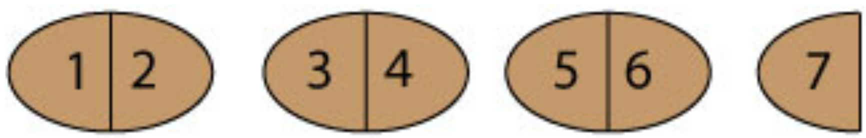
Let's look at one more example just to be sure you have the idea.

> You and some friends sit down to dinner and discover there are $3\frac{1}{2}$ rolls left from your meal the night before. You decided to split all the rolls into $\frac{1}{2}$–size pieces (except for the one that already is) and then divvy them up between you. How many $\frac{1}{2}$–size pieces will there be?

We start with 3 full-size rolls and half of a roll as seen below.



Then, we break the rolls in half and count the total number of $\frac{1}{2}$–size pieces.



From the diagram, we can see easily that are there are 7 pieces of size $\frac{1}{2}$. So, depending on how many friends you have eating with you that night, you can pass out the pieces and maybe keep some extra for yourself. :-0)

What would the mathematics look like for this problem?

Problem	Multiplication Steps	Result	Check
$3\frac{1}{2} \div \frac{1}{2}$	$3\frac{1}{2} \div \frac{1}{2} = \frac{7}{2} \div \frac{1}{2}$ Convert $3\frac{1}{2}$ to an improper fraction $= \frac{7}{\mathbf{2}} \cdot \frac{\mathbf{2}}{1}$ Multiply by reciprocal of $\frac{1}{2}$. $= \frac{7}{1}$ Remove the common factor 2. $= 7$ Simplify to get final result.	$3\frac{1}{2} \div \frac{1}{2} = 7$	$7 \cdot \frac{1}{2} = \frac{7}{1} \cdot \frac{1}{2}$ $= \frac{7}{2} = 3\frac{1}{2}$

Steps to divide fractions (full list):
1. Convert any whole numbers to fractions (over 1).
2. Convert any mixed numbers to improper fractions.
3. Change DIVISION to MULTIPLICATION TIMES THE RECIPROCAL of the SECOND fraction.
4. Multiply straight across.
4. Reduce along the way if possible (only after switching to multiplication).
5. Present final, reduced answer at the end.

NOTE: We do not need to obtain a common denominator when dividing fractions!

Example 5: Divide each of the following. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a) $2 \div \frac{1}{4} =$

b) $2 \div \frac{2}{5} =$

c) $\frac{7}{2} \div \frac{3}{4} =$

d) $\frac{8}{12} \div 4 =$

d) $3\frac{1}{2} \div 5\frac{3}{8} =$

e) $3 \div 2 =$

YOU TRY

6. Divide each of the following. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{2}{3} \div 11 =$

b. $\frac{1}{5} \div 7 =$

c. $3\frac{1}{4} \div \frac{1}{2} =$

APPLICATIONS OF FRACTION DIVISION

Example 7: If part of a recipe for Albondigas Soup calls for 3 small potatoes, $1\frac{1}{2}$ cups of salsa, and 2 pounds of ground beef, how much of each of these ingredients would be needed to make half of the recipe?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

YOU TRY

8. Sally was cutting a large tree into log sections that would fit into her fireplace. If her fireplace would take a log that was $1\frac{1}{4}$ feet long and her tree was 100 feet long, how many sections of $1\frac{1}{4}$ feet length would she cut out of the tree?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

EXPONENTS/ORDER OF OPERATIONS

Remember again our order of operations from Lesson 1? We will use the same order when working with fraction expressions that involve multiple operations and exponents.

P	Simplify items inside Parenthesis ( ), brackets [ ] or other grouping symbols first.
E	Simplify items that are raised to powers (Exponents)
M D	Perform Multiplication and Division next (as they appear from **Left to Right**)
A	Perform Addition and Subtraction on what is left. (as they appear from **Left to Right)**

Example 9: Evaluate. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\left(\frac{3}{4}\right)^3 =$

b. $\frac{3}{5}\left(\frac{2}{3}\right)^2 =$

Example 10: Evaluate. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{3}{4}+\frac{4}{5}\div\frac{2}{3}$

b. $\left(2-\frac{1}{3}\right)^2 \div \left(\frac{1}{4}+\frac{1}{6}\right)$

YOU TRY

11. Evaluate. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\left(\frac{3}{7}\right)^3 =$

b. $\frac{4}{5}\div\left(\frac{2}{3}\right)^2 =$

SPECIAL CASES

Example 12: What happens when you multiply by 0?

$$\left(\frac{3}{4}-\frac{1}{3}\right)\cdot\left(\frac{2}{4}-\frac{1}{2}\right)$$

Example 13: What happens when you divide by 0?

$$\frac{2}{3}\div\frac{0}{1}$$

YOU TRY

14. Bill earns $10 for every hour he works each week up to 40 hours. Any additional hours are considered overtime and he earns "time and a half" wages. If he worked 56 hours one week, what were his total earnings?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

LESSON 4 – PRACTICE PROBLEMS

1. Multiply and simplify. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{1}{6}\cdot\frac{3}{5}$ b. $\frac{8}{9}\cdot\frac{9}{12}$ c. $\frac{3}{4}\cdot 0$

d. $1\frac{1}{2}\cdot\frac{1}{2}$ e. $3\frac{1}{3}\cdot 2\frac{2}{5}$

2. Find the reciprocal for each of the following (if possible).

a. $\frac{1}{3}$ b. 2 c. $\frac{22}{7}$

d. $3\frac{1}{2}$ e. $\frac{0}{1}$

3. Divide and simplify. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $3\div\frac{1}{3}$ b. $\frac{2}{5}\div\frac{5}{2}$ c. $\frac{1}{4}\div 6$

d. $3\frac{1}{2}\div 1\frac{1}{3}$ e. $\frac{2}{4}\div\frac{1}{2}$

4. Perform the indicated operations and simplify. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{3}{4} \div \frac{4}{5} \cdot \frac{5}{6}$

b. $\frac{1}{2} - \frac{1}{3} \cdot \frac{1}{4}$

c. $\left(2 - \frac{8}{5}\right)^2$

d. $1 - \left(\frac{1}{2}\right)^2$

e. $\frac{1}{2} \div \frac{3}{4} \div \frac{1}{4}$

5. Solve each of the following application problems using the 5-step process illustrated in the lesson. Leave final answers in mixed number form if possible.

a. Suppose your school costs for this term were \$2500 and financial aid covered $\frac{3}{4}$ of that amount. How much did financial aid cover?

b. If, on average, about $\frac{4}{7}$ of the human body is water weight how much water weight is present in a person weighing 182 pounds?

c. If, while training for a marathon, you ran 920 miles in $3\frac{1}{2}$ months, how many miles did you run each month? (Assume you ran the same amount each month)

d. If the area of a rectangle is given by the formula $A = L \cdot W$ where L = length and W = width, compute the amount of carpet needed for a rectangular room that is $12\frac{1}{2}$ feet long and $14\frac{1}{4}$ feet wide. Your final units will be square feet.

e. A recipe for Albondigas Soup makes 6 servings. A partial ingredient list includes: 1 quart of water, $\frac{1}{3}$ cup milk, 2 beef bouillon cubes, and 4 large carrots. How much of each ingredient would be in each serving? Reduce each fraction and use mixed numbers where appropriate.

LESSON 4 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Multiply and simplify. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{2}{3}\cdot\frac{4}{7}$

b. $\frac{5}{3}\cdot\frac{7}{10}$

c. $5\frac{2}{3}\cdot 4\frac{5}{7}$

2. Find the reciprocal for each of the following (if possible).

a. $\frac{2}{5}$

b. $\frac{7}{3}$

c. $4\frac{1}{2}$

3. Divide and simplify. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{5}{7}\div\frac{1}{7}$

b. $4\div\frac{1}{2}$

c. $8\frac{2}{3}\div 3$

d. $2\frac{1}{3}\div 6\frac{3}{7}$

4. Perform the indicated operations and simplify. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{7}{8}-\left(\frac{3}{4}\right)^2$

b. $\frac{5}{7}\div\frac{6}{11}\cdot\frac{2}{5}$

5. Solve the following problem using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

On your first math test, you earned 75 points. On your second math test, you earned $\frac{6}{5}$ as many points as your first test. How many points did you earn on your second math test?

6. Solve the following problem using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

You are serving cake at a party at your home. There are 12 people in total and $2\frac{3}{4}$ cakes. (You ate some before they got there!). If the cakes are shared equally among the 12 guests, what fraction of a cake will each guest receive?

LESSON 5 - DECIMALS

INTRODUCTION

Now that we know something about whole numbers and fractions, we will begin working with types of numbers that are extensions of whole numbers and related to fractions. These numbers are called *decimals* or *decimal numbers*.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Identify *decimal place values*.	1, YT4c
Write *decimal numbers* in words.	2, YT4a
Round *decimals numbers* to a given place value.	3, YT4b
Convert *decimals* to fractions.	5, YT6
Convert fractions to *decimals*.	7, YT8
Simplify *decimal* expressions with the aid of a calculator	9, YT10
Solve problems involving *money decimals*.	11, 12, YT13
Order decimals & fractions from least to greatest.	14, YT16
Solve applications involving *decimals*.	15, YT17

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Decimals/Decimal Numbers
- Decimal Point
- Decimal Place Values
- Rounding Decimals
- Mathematical Operations
- Order of Operations (PEMDAS)
- Money Decimals
- Ordering Decimals

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

DECIMALS, PLACE VALUE, and ROUNDING

*Decimal*s contain numbers to the right of the *decimal point*. The place value chart below identifies the first few decimal places. Use this chart to help you with the examples below.

BILLIONS			,	MILLIONS			,	THOUSANDS			,	ONES			.	DECIMALS			
100	10	1	,	100	10	1	,	100	10	1	,	100	10	1	.	tenths	hundredths	thousdandths	ten-thousandths

Example 1: What place does the DIGIT 4 occupy in each number?

a. 324, 231.17

b. 256.134

c. 0.04

d. 1.4671

Example 2: Write in words the numbers listed in Example 1.

a. 324, 231.17 In Words: ____

b. 256.134 In Words: ____

c. 0.04 In Words: ____

d. 1.4671 In Words: ____

As in lesson 1, *rounding* is used to approximate numbers to a particular place value. The process of rounding involves choosing a number (to the indicated place value) that is closest to the number you have. *Decimal rounding* is similar to whole number rounding, however, the decimal place values have different names and locations.

Example 3: Round each of the following numbers to the indicated place value.

a. 42.3456 to the nearest tenths place

b. 42.3999 to the nearest hundredths place

c. 42.3456 to the nearest thousandths place

YOU TRY

4a. Write the number 12.619 using words.

____

4b. Round 12.699 to the nearest hundredth. ______________________

4c. What place does the digit 6 occupy in the number 12.619? ____________

CHANGING FROM DECIMALS TO FRACTIONS

Decimals are really fractions in disguise, as you will see in the examples below.

Example 5: Change each of the following to a simplified fraction or mixed number.

a. 0.6 b. 1.15 c. 0.0564

YOU TRY

6. Change each of the following to a simplified fraction or mixed number.

a. 5.375 b. 0.025

Your calculator can help you convert decimals to fractions. Look for Frac<->Dec somewhere on the calculator. Refer to your calculator manual for steps.

CHANGING FROM FRACTIONS TO DECIMALS

Fractions can easily be converted to decimals using the mathematical operation of division.

Example 7: Change each of the following to a decimal. Round to the nearest hundredth as appropriate.

a. $\frac{3}{4}$

b. $\frac{52}{10}$

c. $\frac{1}{3}$

d. $10\frac{3}{7}$

YOU TRY

8. Change each of the following to a decimal. Round to the thousandths place as appropriate.

a. $\frac{531}{25}$

b. $\frac{41}{9}$

c. $3\frac{6}{11}$

OPERATIONS WITH DECIMALS – CALCULATOR ASSISTED

When performing the mathematical operations of addition, subtraction, multiplication, and division using decimals, our calculator is a great support tool. Once the given numbers are combined, rounding often comes into play when presenting the final result.

Example 9: Use your calculator to compute each of the following. Round as indicated.

a. Multiply $4.32 \cdot 3.17$ then round the result to the nearest tenth.

b. Divide $523.14 \div 23.56$ then round the result to the nearest thousandth.

c. Multiply $(0.1)^2$. Write your result first in decimal form. Then, convert to a simplified fraction.

d. Combine the numbers below. Round your final result to the nearest whole number.

$3.721 + 4.35 \cdot 21.72 - 0.03$

YOU TRY

10. Use your calculator to combine the numbers below. Round your final result to the nearest hundredth. When computing, try to enter the entire expression all at once.

$(6.41)^2 - 5.883 \div 2.17$

DOLLARS AND CENTS – WORKING WITH MONEY

Pennies = Cents = 2 decimal places = Hundredths place

Dollars = ones place

Example 11: Write each of the following word phrases as a decimal

a. Twelve dollars and seventy-five cents

b. Thirty-two cents

c. Five cents

d. One hundred dollars and seven cents

Example 12: Round each of the following monetary amounts as indicated:

a. $127.56 to the nearest dime

b. $127.56 to the nearest dollar

c. $127.56 to the nearest ten dollars

d. $127.56 to the nearest hundred dollars

YOU TRY

13a. Write as a decimal: Twenty dollars and five cents ______________________________

13b. Round $311.58 to the nearest dollar. ______________________________

ORDERING DECIMALS & FRACTIONS

When given numbers in decimal and/or fraction form, can you order them correctly from smallest to largest? The following examples will explain ways to do that.

Example 14: Order each of the following sets of numbers from smallest to largest.

a. 0.042, 0.420, 0.402

b. 1.73, $1\frac{11}{15}$, 1.7

APPLICATIONS WITH DECIMALS

Example 15: In preparation for mailing a package, you place the item on your digital scale and obtain the following readings: 6.51 ounces, 6.52 ounces, and 6.60 ounces. What is the average of these weights? Round to the nearest hundredth of an ounce.

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

YOU TRY

16. Order the following set of numbers from smallest to largest. Show work or explain your reasoning.

$$3.555,\quad 3.055,\quad 3.55,\quad 3\frac{3}{5},\quad 3.5,\quad 3.05$$

17. Rally went to Target with \$40 in his wallet. He bought items that totaled \$1.45, \$2.15, \$7.34, and \$14.22. If the tax comes to \$2.26, how much of his \$40 would he have left over? Round to the nearest cent.

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

LESSON 5 - PRACTICE PROBLEMS

1. Name the place value of the "8" digit in each of the following numbers.

   a. 183.234

   b. 234.183

   c. 30.816

   d. 0.00854

   e. 1.0008

2. Write in words the name of each of the following numbers.

   a. 183.234

   b. 234.183

   c. 30.816

   d. 0.00854

   e. 1.0008

3. Round each number to the given decimal place.

   a. 156.247 to the nearest hundred

   b. 156.247 to the nearest hundredth

   c. 23.4999 to the nearest hundredth

   d. 23.4035 to the nearest thousandth

   e. 21.512 to the nearest whole number

4. Convert each decimal to a fraction or mixed number as appropriate.

   a. 0.05 b. 1.34 c. 2.006

   d. 0.125 e. 1.2

5. Convert each fraction to a decimal. Round to the hundredths place as appropriate.

a. $1\frac{1}{2}$ b. $\frac{22}{3}$ c. $\frac{105}{23}$

d. $\frac{25}{100}$ e. $\frac{3}{7}$

6. Add or subtract each of the following using your calculator. Round each result both to the *hundredths place* and to the nearest *whole number*. Be sure you round your initial computation to these places…don't take your hundredths rounding and round to the nearest whole number.

a. 301.25 + 21.456

b. 14.256 - 0.0132

c. 5 + 6.238

d. 1.256 - 0.34

e. 125.543 + 1.23

7. Multiply or divide each of the following using your calculator. Round each result both to the *hundredths place* and to the nearest *whole number*. Be sure you round your initial computation to these places…don't take your hundredths rounding and round to the nearest whole number.

a. $301.25 \cdot 21.456$

b. $14.256 \div 0.0132$

c. $5 \cdot 6.238$

d. $0.256 \div 0.34$

e. $125.543 \cdot 1.23$

8. Use correct order of operations for each of the following. You may use your calculator but you should show the intermediate simplification steps. Round your final result to the nearest hundredths place as appropriate.

a. $(4.01)^2 - 2.25 \cdot 3.85$

b. $(3.523 - 1.20)^2 + 4.0 - 2.14$

c. $12.82 \cdot 6.238 + 3.457 - 5.02 \cdot 6.83712$

d. $0.256 \div 0.34 \cdot 7.813 - (0.214)^2$

e. $(2.1)^3 - (0.15 + 0.19)^2$

9. Order each of the following from largest to smallest.

a. 0.1, 0.01, 0.11

b. 2.3, 2.33, 2.03

c. $\frac{1}{2}$, 0.501, 0.05

d. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

e. $\frac{6}{7}, \frac{7}{8}, \frac{5}{6}$

10. Solve each of the following application problems using the 5-step process illustrated in the lesson. Leave final answers in a form appropriate for the problem. Round decimals to hundredths place as appropriate.

a. Sarah Smartshopper receives ten cents off per gallon on gas for every $100 she spends at the grocery store during a given month. During the month of October, he spent $45.23, $102.34, $13.67, $34.56, $48.72, and $52.12. What will Sarah's gas discount be for October?

b. Wendy Watersaver just received her monthly water usage data from her local water department. For the past 6 months, her water used (in thousands of gallons) was 19.9, 25.6, 28.8, 22.5, 20.3, and 19.2. What was her average usage during this time? (Round to the nearest tenth)

c. Marty Mathwhiz is standing in line at the store with his friend Danny Doubter. Marty says that he can estimate his purchase, without using a calculator, within 50 cents of the actual amount. Danny, of course, did not believe him. Marty bought items in the amounts of $1.25, $2.04, $5.62, $8.81, $6.12, and $12.99. Marty estimated his items at $37. First of all, was he within the 50 cent limit for his estimation and second, how might he have accomplished this?

d. Henrietta Hardworker normally earns $8.50 per hour in a given 40-hour work-week. If she works overtime, she earns time and a half pay per hour. During the month of October, she worked 40 hours, 50 hours, 45 hours, and 42 hours for the four weeks. How much did she earn total for October?

e. Chris Carpenter is making a gazebo for his yard. He has a piece of wood that is 13 feet long and he needs to cut it into pieces of length 5.3 inches. How many pieces of this size can he cut from the 13 foot piece of wood?

LESSON 5 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Name the place value of the digit "5" in the following number.

   17.19524

2. Write the name of the following number in words.

   823.029

3. Round each number to the given decimal place.

   a. 263.413 to the nearest hundred

   b. 263.413 to the nearest hundredth

   c. 57.1349 to the nearest hundredth

   d. 57.1349 to the nearest thousandth

   e. 623.567 to the nearest whole number

4. Convert the decimal to a fraction.

   0.65

5. Convert the decimal to a mixed number.

   72.345

6. Convert the fraction to a decimal. Round to the nearest hundredth as needed.

$$\frac{7}{9}$$

7. Convert the mixed number to a decimal. Round to the nearest hundredth as needed.

$$11\frac{2}{7}$$

8. Multiply or divide each of the following using your calculator. Round each result both to the *hundredths place* and to the nearest *whole number*. Be sure you round your initial computation to these places…don't take your hundredths rounding and round to the nearest whole number.

a. $136.204 \cdot 52.09$ b. $873.24 \div 42.3$

9. Order each of the following from smallest to largest.

a. 5.23, 5.65, 5.234, 5.3 b. $\frac{2}{7}, \frac{2}{11}, \frac{2}{9}, \frac{2}{5}$

10. Solve the following problem using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

a. Callie ordered 4 items online. She is charged \$2.37 per pound per shipping. The items weighed 3.2 lbs., 4.6 lbs., 9.2 lbs. and 1.5 lbs. How much will be charged for shipping? (Round to the nearest cent).

b. Penny is making barrettes for her online business. Each barrette needs 2.3 inches of ribbon. If Penny has 4 feet of ribbon, how many barrettes can she make?

c. Glen visits the grocery store once a week for groceries. The amount he spent on five separate visits was \$52.35, \$36.93, \$44.79, \$88.98, \$55.22. What is the average amount Glen spent per week over these five weeks?

LESSON 6 - PERCENTS

INTRODUCTION

So far in this course we have worked with different types or forms of numbers including whole numbers, fractions, and decimals. With each number type or form we have learned to join them using mathematical operations or to convert them from one form to another. In this lesson, we will combine our knowledge of whole numbers, decimals, and fractions and apply this knowledge to learning about the concept of *percent (%).*

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Convert *percent* to decimal	2, YT3
Convert *percent* to fraction	2, YT3
Convert decimal to *percent*	2, YT3
Convert fraction to *percent*	2, YT3
Solve equations of the form $x = a \cdot b$	WE4
Solve equations of the form $a \cdot x = b$	WE5
Create and solve *percent* equations: Type I	6, YT7
Create and solve *percent* equations: Type II	8, YT9
Create and solve *percent* equations: Type III	10, YT11
Solve applications involving *percent*	1, 12-14, YT15
Determine *percent increase* or *decrease*	16, 17, YT18
Determine *percent discount*	19, YT21a
Compute *simple interest*	20, YT21b

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Percent
- Variable
- Percent Equations (Type I, II, III)
- Percent Increase or Decrease
- Percent Discount
- Simple Interest
- Principle
- Interest Rate

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

DEFINITON OF PERCENT

Numbers written in percent form represent amounts out of 100. The word "*percent*" actually means "per 100" (Think of it as "per cent" and there are 100 cents in $1). The following example will help us start thinking about numbers in percent form.

Example 1: At a recent "Rats on Rafts" rock concert (there actually is a 2012 band with this name! ☺), 50% of the attendees were under 18, 25% were 18 – 24 and the rest were over 24. If 22,140 people attended the concert, how many were in each age group?

DECIMALS, FRACTIONS, PERCENTS

Decimals, fractions, and percents are closely connected. The following table shows how to convert from one type to the other.

Percent to Decimal	$50\% = .50$	Remove % sign. Divide by 100.
Percent to Fraction	$50\% = \frac{50}{100} = \frac{1}{2}$	Remove % sign. Place over 100. Reduce fraction.
Decimal to Percent	$0.50 = 50\%$	Multiply by 100. Include % sign.
Fraction to Percent	$\frac{1}{2} = .50 = 50\%$	Divide. Multiply by 100. Include % sign.

Example 2: Complete the missing parts of the table. Round to THREE decimal places as need. Simplify all fractions. Show all work.

Fraction	Decimal	Percent
		32%
	0.040	
$\frac{3}{4}$		
	0.625	
		150%
$1\frac{3}{7}$		

YOU TRY

3. Complete the missing parts of the table. Round decimal part to FOUR decimal places as needed. Simplify all fractions. Show all work.

Fraction	Decimal	Percent
a. $\frac{1}{9}$		
b.	0.0625	
c.		80%

Now that we have a feeling for what percent numbers look like and how they can be written as decimals or fractions, we will learn how to compute percent numbers given different pieces of information. Before we do that, however, we need to learn one new idea that will make working with percent numbers a fairly straightforward process.

SOLVING EQUATIONS OF THE FORM x = a · b

Worked Example 4 In mathematics, we let *variables* (letters) take the place of number values that we do not know. The mostly commonly used variable is the variable "x". In each of the following, to determine the value of the unknown number (x), we simply multiply the two numbers that we do know.

a. $x = 3 \cdot 4$, $x = 12$

b. $x = 2.5 \cdot 8$, $x = 20$

c. $x = 0.12 \cdot 50$, $x = 6$

SOLVING EQUATIONS OF THE FORM a · x = b

These problems are a little different in that there is a division step in each one. Follow the examples carefully and notice the check for accuracy. In the first, three, you could probably determine the unknown x without having to divide (for example, what number times 3 gives 6? Has to be 2). But the last three are more complicated and involve some decimal results.

Worked Example 5 Determine the value of the unknown number x. Show complete division steps and also show a check. Round to two decimals as needed. [*Note: There are also video links for this problem!]*

a. $3x = 6$, $x = \frac{6}{3}$, $x = 2$	b. $12x = 24$, $x = \frac{24}{12}$, $x = 2$	c. $2x = 20$, $x = \frac{20}{2}$, $x = 10$
Check: $3 \cdot 2 = 6$, $6 = 6$	Check: $12 \cdot 2 = 24$, $24 = 24$	Check: $2 \cdot 10 = 20$, $20 = 20$
d. $\frac{1}{4}x = 9$, $x = 9 \div \frac{1}{4} = 9 \cdot 4$, $x = 36$	e. $5x = 12$, $x = \frac{12}{5}$, $x = 2.40$	f. $6x = 11$, $x = \frac{11}{6}$, $x = 1.83$
Check: $\frac{1}{4} \cdot 36 = 9$, $\frac{36}{4} = 9$, $9 = 9$	Check: $5 \cdot \frac{12}{5} = 12$, $\frac{60}{5} = 12$, $12 = 12$	Check: $6 \cdot \frac{11}{6} = 11$, $\frac{66}{6} = 11$, $11 = 11$

CREATING AND SOLVING PERCENT EQUATIONS

When working with a problem involving percent, the most straightforward way to solve it is by setting up a *percent equation*. The information we just covered on the previous page will help solve percent equations once they are set up. The information below provides translation guidance for words or phrases that are part of percent statements.

- The percent (usually represent as a decimal)
- Multiplication (replace the word "of" with multiplication)
- The unknown (usually represented by the word "what" and replace with "x")
- Equals (replace the word "is" with "=")

Let's see how these translations are used when setting up the three main types of percent problems.

TYPE I: Unknown is A% of B

Example 6: Determine the missing number in each of the following. Round to two decimals as needed.

a. What is 12% of 20?

b. What is 30.45% of 450?

c. 12% of 600 is what number?

d. What number is 0.5% of 8?

YOU TRY

7. Determine the missing number in each of the following. Round to two decimals as needed.

a. What is 15% of 324?

b. 25.12% of 132 is what number?

TYPE II: A% of Unknown is B

Example 8: Determine the missing number in each of the following. Round to TWO decimals as needed. Show all work.

a. 60% of what number is 15?

b. 25 is 12.25% of what number?

c. 175% of what number is 325.16?

d. 20 is 0.14% of what number?

YOU TRY

9. Determine the missing number in each of the following. Round to TWO decimals as needed. Show all work.

a. 40% of what number is 20?

b. 105 is 15.15% of what number?

TYPE III: Unknown % of A is B

Example 10: Determine the missing number in each of the following. Round to TWO decimals as needed. Show all work.

a. What % of 140 is 3.8?

b. What percent of 620 is 136.4?

c. What % of 25 is 0.05?

d. 240 is what percent of 100?

YOU TRY

11. Determine the missing number in each of the following. Round to TWO decimals as needed. Show all work.

a. What % of 12 is 8?

b. 105 is what percent of 123?

APPLICATIONS OF PERCENTS – TYPES I, II, III

Try to recognize the percent problem as one of our three types and set up the percent equation to solve for the missing part. Use a modified version of our problem-solving process by circling the given information and underlining the goal in each problem.

Example 12: (TYPE I)
At a restaurant, the bill comes to $51.23. If you decide to leave a 14% tip, how much is the tip and what is the final bill? Round to the nearest cent.

Example 13: (TYPE II)
Joyce paid $33.00 for an item at the store that was marked as 45 percent off the original price. What was the original price? Round to the nearest cent.

Example 14: (TYPE III)
Trader Joe's sold 8233 bags of tortilla chips recently. If 5178 of these bags were fat free, find the percent that were fat free. Round your answer to the nearest whole percent.

YOU TRY

15. To win the election as president of the United States of America, a person must obtain 270 out of 538 possible votes from the electoral college. What percentage of the overall electoral votes is this? Round to 4 decimals initially. Be sure to set up your percent statement and equation as in the examples.

PERCENT INCREASE OR DECREASE

Example 16: Determine the percent increase or decrease for the change from 20 to 30. Round % to the nearest whole number.

Example 17: Determine the percent increase or decrease for the change from 30 to 20. Round % to the nearest hundredth.

YOU TRY

18. Determine the percent increase or decrease for each of the following. Round % to the nearest hundredth.

a. the change from 54 to 62

b. the change from 50 to 40

% DISCOUNT

Example 19: A $725 couch is on sale for 20% off. Find the amount of the discount and the sale price. Round to the nearest cent. [Circle GIVENS and underline GOAL]

SIMPLE INTEREST

SIMPLE INTEREST = PRINCIPLE x INTEREST RATE x TIME

In shortened form, $I = PRT$

Remember the following:

- Principle is the amount borrowed
- Interest Rate is the annual rate and should be written as a decimal
- Time is in years

Example 20: Calculate the simple interest and the final balance on $200 borrowed at 8% interest over 4 months.

YOU TRY

21a. An $85 pair of sunglasses is on sale for 30% off. Find the amount of the discount and the sale price. Round to the nearest cent. [Circle GIVENS and underline GOAL]

21b. Calculate the simple interest and the final balance on $14,000 borrowed at 3% interest over 10 years. Round to the nearest cent. [Circle GIVENS and underline GOAL]

LESSON 6 – PRACTICE PROBLEMS

1. Complete the missing parts of the table. Round to THREE decimal places as need. Simplify all fractions. Show all work.

Fraction	Decimal	Percent
$\frac{3}{4}$		
	0.375	
		72%
	1.3	
$\frac{2}{7}$		

2. Solve each of the following for x. Round to the hundredths place.

   a. $x = 0.15 \cdot 30$

   b. $x = 0.075 \cdot 100$

   c. $x = 1.5 \cdot 40$

   d. $x = 0.128 \cdot 17.45$

   e. $x = 1.43 \cdot 12$

3. Solve each of the following for x. Round to the hundredths place.

a. $75x = 5$

b. $12x = 3.25$

c. $14x = 5.03$

d. $125x = 42$

e. $\frac{1}{3}x = 15$

4. Determine the missing number in each of the following. Round to two decimals.

a. 6% of what number is 12?

b. 82% of what number is 116?

c. 123% of what number is 25?

d. 20 is 0.18% of what number?

e. 120 is 125% what number?

5. Determine the missing number in each of the following. Round to two decimals.

a. What is 5% of 25?

b. 0.01% of 12 is what number?

c. 123% of 100 is what number?

d. 12.56% of 72 is what number?

e. 50% of 127 is what number?

6. Determine the missing number in each of the following. Round to two decimals.

   a. What % of 25 is 5?

   b. 12 is what percent of 40?

   c. What percent of 32 is 48?

   d. 15 is what percent of 23?

   e. 0.25 is what percent of 3?

7. Determine the percent increase or decrease for the change for each of the following:

   a. 12 to 15

   b. 22 to 18

   c. 30 to 60

   d. 120 to 90

   e. 90 to 100

8. Solve each of the following application problems using the methods from this lesson.

   a. In a recent poll, 28% of the 750 individuals polled indicated that they would vote purely Democratic in the next election. How many of the individuals would vote a straight Democratic ticket?

b. If you decrease your daily intake of calories from 2500 to 1750, by what percent do your daily calories decrease?

c. On a recent trip to Walmart, you bought $75.25 worth of goods and paid a total of $82.02. What was the rate of sales tax that you paid?

d. If you invest $5000 at simple interest of 8% per year for 6 years, how much money will you earn from interest? How much money will you have at the end of 6 years?

e. In the U.S. Civil War, 750,000 people were estimated to have died. If that number represented 2.5% of the U.S. population of the day, how many people lived in the U.S. during the Civil War? If a war of that scale happened today and the same percentage of people died, how many people would be killed (assume U.S. population of 314,721,724 people). [Source: Smithsonian Magazine, November 2012, page 48]

LESSON 6 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Complete the missing parts of the table. Round to THREE decimal places as need. Simplify all fractions. Show all work.

	Fraction	Decimal	Percent
a	$\frac{3}{5}$		
b		1.24	
c			16%

2. Determine the missing number. Round to two decimals as needed.

26% of what number is 15?

3. Determine the missing number. Round to two decimals as needed.

0.23% of 37 is what number?

4. Determine the missing number. Round to two decimals as needed.

25 is what percent of 13?

5. Determine the percent increase or decrease for the change for each of the following. Round to two decimals as needed.

a. 32 to 48

b. 74 to 23

6. Solve the following problem using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

a. Sara had a party for her parent's anniversary. Fifty-six people attended. This was approximately 72% of the people she invited. How many people did Sara invite? (Round to the nearest person)

b. Amy decreased her restaurant spending from $287 a month to $54 a month. What percent decrease is this?

c. Jose spent $136.25 on a video game including 9% sales tax. What was the cost of the video game without tax?

LESSON 7 – RATIOS, RATES, & PROPORTIONS

INTRODUCTION

In Lessons 2, 3, & 4, we spent time building a base of knowledge about fractions and operations with fractions. In this lesson, we will expand on that knowledge and spend time learning about and solving problems with a special kind of fraction called a *ratio.*

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Write and simplify *ratios*	1, 2, YT5a
Write and simplify *rates*	3, 4, YT5b
Compute *unit rates*	6, 7, 8, YT9
Solve proportions using *cross-products*	10, 11, 12, YT13
Solve applications using *proportional reasoning*	14, YT15

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Ratio
- Rate
- Unit Rate
- Proportion
- Cross Product
- Cross Product Method
- Proportional Reasoning

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

RATIOS & RATES

Write a *ratio* to compare two different quantities. Units are important and are always included if they are present to begin with. The examples below demonstrate the different notations you may see when writing ratios.

Example 1: Write "8 feet to16 feet" as a ratio in simplest form.

Example 2: Write "6:18" as a ratio in simplest form.

If the quantities you are comparing have different units, then your ratio is known as a *rate.* Units are especially important here and should absolutely be included.

Example 3: Write "12 miles in 10 hours" as a ratio in simplest form.

Example 4: In a small bag of mixed nuts, 15 were peanuts, 20 were almonds, and 5 were Brazil nuts. Write the ratio of peanuts to almonds in simplest form.

Note: With ratios, the units will cancel out. With rates, the units will not cancel out.

YOU TRY

5. Use the information to write a ratio in simplest form. Indicate if the ratio is also a rate.

a. 5 feet:10 feet

b. 12 geese to 15 ducks

UNIT RATES

A *unit rate* is a special kind of rate in which the denominator of the ratio is equal to 1. This kind of rate allows for easier comparison of different rates as seen in the example below. As with rates, units are essential and must be included.

Example 6: Which is faster, "12 miles in 10 hours" or "10 miles in 8 hours"? Use unit rates to compare.

Example 7: Determine which bag of Cheetos is the better buy.

Bag A: $4.99 for 20.50 oz   Bag B: $4.29 for 12.50 oz

Example 8: Write each of the following as a unit rate:

a. There are 5280 feet in a mile

b. There are 60 seconds per each minute

c. Gasoline costs $3.49 a gallon

YOU TRY

9. Amazon.com recently advertised the following choices for ibuprofen tablets (200mg). Use unit rates to determine which is the better buy.

Option 1: 360 pills for $15.45   Option 2: 300 pills for $12.98

PROPORTIONS & PROPORTIONAL REASONING

In Example 3, we were given the rate, "12 miles in 10 hours" which we simplified to "6 miles in 5 hours". Let's see how we might write that as a formal mathematical statement of equality:

$$\frac{12 \text{ miles}}{10 \text{ hours}} = \frac{6 \text{ miles}}{5 \text{ hours}}$$

The statement above is called a *proportion* because it sets two rates (or ratios) equal to each other. Because the above rates are equivalent, the equality statement is true.

Suppose, however, that the following problem was posed:

"If George walks 6 miles in 5 hours, how far would he walk in 10 hours?"

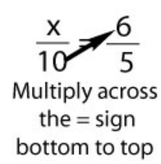
We will use the concept of *variable* from Lesson 6 to set up the following *proportion:*

$$\frac{x \text{ miles}}{10 \text{ hours}} = \frac{6 \text{ miles}}{5 \text{ hours}}$$

The distance George walks in 10 hours is our unknown value and is represented by the *variable x*. Technically, in this problem, we know that our solution for x is 12. But how would we determine that? First, because our ratios of units are the same (miles/hours) we can simplify our statement this way:

$$\frac{x}{10} = \frac{6}{5}$$

Then, we can use one *cross-product* to rewrite as follows:



$$x = \frac{6 \cdot 10}{5}$$

And finally, we can write our final solution as $x = \frac{60}{5} = 12$.

The final answer to our original question, "if George walks 6 miles in 5 hours, how far would he walk in 10 hours" is that George could walk 12 miles in 10 hours. We solved this problem using *proportional reasoning,* one of the most-used problem solving techniques in mathematics.

The following examples will illustrate additional ways to work with and solve proportions using the *cross-product method*.

Example 10: Use the cross-product method to determine the value for t in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{3}{4} = \frac{t}{40}$

b. $\frac{t}{2} = \frac{3}{5}$

Example 11: Use the cross-product method to determine the value for x in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{6}{12} = \frac{18}{x}$

b. $\frac{2.3}{x} = \frac{4.1}{5.6}$

Example 12: Use the cross-product method to determine the value for r in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{r}{5} = 3$

b. $\frac{1/2}{4} = \frac{8}{r}$

YOU TRY

13. Solve the proportions showing all possible steps. Round your answer to the nearest hundredth as needed.

a. $\frac{x}{12}=\frac{3}{6}$

b. $\frac{6}{5}=\frac{10}{p}$

APPLICATIONS OF PROPORTION

Example 14: Ten gallons of water leak from a hose in 20 hours. At this rate, how much water will leak in 10 days? Practice circling the GIVENS and underlining the GOALS to start your problem-solving process.

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

YOU TRY

15. Mary earned $112.50 last week working 12 hours at her part-time job. If she works 15 hours this week and is paid the same rate, how much will she earn? Use proportional reasoning to determine your result. Round to the nearest cent.

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

LESSON 7 - PRACTICE PROBLEMS

1. Write each ratio in simplest form.

a. 3 to 7

b. 4:12

c. $\dfrac{12 \text{ inches}}{24 \text{ inches}}$

d. 14 to 42

e. $\dfrac{16}{42}$

2. Write each rate in simplest form.

a. 30 miles in 4 hours

b. 24 inches to 4 feet

c. 12 boys to 18 girls

d. 18 cars to 32 bicycles

e. 15 men to 35 women

3. Write the unit rate for each of the following. Round to two decimals.

   a. 150 miles in 3 hours

   b. 24 inches to 2 feet

   c. $18.25 for 4 gallons

   d. $1.45 for 6 ounces

   e. 74 pounds per 12 square inches

4. Solve each of the following proportion problems for the given variable. Round to two decimals as needed.

a. $\frac{x}{2} = \frac{12}{4}$

b. $\frac{r}{5} = \frac{6}{3}$

c. $\frac{10}{6} = \frac{x}{12}$

d. $\frac{1.5}{3} = \frac{t}{6}$

e. $\frac{x}{2.4} = \frac{3.8}{3.04}$

5. Solve each of the following proportion problems for the given variable.
Round to two decimals as needed.

a. $\frac{3}{x} = \frac{2}{4}$

b. $\frac{3}{r} = \frac{3}{6}$

c. $\frac{6}{10} = \frac{12}{x}$

d. $\frac{7}{3} = \frac{14}{t}$

e. $\frac{2.5}{x} = \frac{4}{8}$

6. Solve each of the following proportion problems for the given variable.
Round to two decimals as needed.

a. $\frac{x}{2} = 6$

b. $t = \frac{8}{4}$

c. $\frac{1/3}{6} = \frac{x}{12}$

d. $\frac{1.5}{1/3} = \frac{t}{6}$

e. $\frac{2.4}{x} = 5$

7. Solve the following problems using the 5-step problem solving process. Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

a. If the scale on a map is 1 inch to 20 miles, what is the actual distance between two towns that are 3 inches apart on the map?

b. In November 2012 President Obama visited Phnom Penh, Cambodia as part of a summit of Asian leaders. Traffic in the city came to almost a complete standstill with cars moving at a rate of 2 miles in 4 hours. At this rate, how long would it take to travel a distance of 3.5 miles?

c. Ryan works a part-time job mowing lawns and can easily mow 3 lawns in 5 hours. If he got very busy one day and mowed 7 lawns, how long did it take him?

d. The director of a day care center can feed 7 children lunch for a week with 4 pounds of macaroni and cheese. If she has 16 pounds of macaroni and cheese, how many children can she feed lunch for a week?

e. On Thanksgiving Day, 2012, the New England Patriots scored 35 points in the second quarter to aid in their eventual 49 - 19 victory over the New York Jets. What was their unit rate of scoring per minute during that quarter (15 minutes)? If they had continued the second half (two more quarters) at that rate, what would the final score have been (New England scored 0 points in the first quarter).

LESSON 7 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Write each ratio in simplest form.

a. 5 to 10 b. 8:14 c. $\dfrac{6\,feet}{4\,feet}$

2. Write each rate in simplest form.

a. 60 miles in 2 hours b. 36 inches to 3 feet c. 14 men to 21 women

3. Write the unit rate for each of the following. Round to the nearest hundredth.

a. 180 miles in 3 hours b. $25.00 for 6 gallons c. 86 pounds per 16 square inches

4. Solve each of the following proportion problems for the given variable.
Round to two decimals as needed.

a. $\dfrac{x}{4}=\dfrac{5}{12}$ b. $\dfrac{r}{5}=\dfrac{21}{7}$ c. $\dfrac{10}{8}=\dfrac{x}{32}$

5. Solve each of the following proportion problems for the given variable.
Round to two decimals as needed.

a. $\dfrac{9}{x}=\dfrac{3}{8}$ b. $\dfrac{5}{r}=\dfrac{3}{8}$ c. $\dfrac{7}{10}=\dfrac{28}{x}$

6. Solve each of the following proportion problems for the given variable.
Round to two decimals as needed.

a. $\frac{1/4}{8} = \frac{x}{12}$ b. $\frac{3.5}{1/2} = \frac{t}{8}$ c. $\frac{6.7}{x} = 8$

7. Solve the following problems using the 5-step problem solving process.
Include the categories: Given, Goal, Math work, Check and Final result as a complete sentence.

a. Ray can bike 25 miles in 3 hours. At this rate, how long will it take him to bike 43 miles?

b. Julia can feed 8 relatives with a 12 pound turkey. How many pounds of turkey does she need to feed 14 relatives?

LESSON 8 - STATISTICS

INTRODUCTION

In this lesson, we will learn the basic language and concepts related to a branch of mathematics that deals with collecting, organizing, and interpreting data. This branch of mathematics is called *statistics*. In addition, the word *statistics* is often used to denote the data and information that are being collected and interpreted.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Define and compute three different *measures of central tendency*	1, YT2
Define and compute *weighted averages*	3, 4, YT5
Discuss data *variability* and compute the *range* of a data set	6, 7, YT8
Compute *measures of central tendency* with data sets that contain *outliers*	9
Use *tables and graphs* to interpret and analyze data	10, 11, YT12, 13, YT14

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Average
- Measures of Central Tendency
- Mean
- Median
- Mode
- Weighted Average
- Variation
- Range
- Outlier

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

MEASURES OF CENTRAL TENDENCY

When we are given a set of data points, particularly if that set is very large, we want to get a feel for the data by getting a sense of what single number most accurately represents that data. To do that, we compute one or more of the following *Measures of Central Tendency* or *Averages*.

- *Mean* is the sum of a set of values divided by the number of values.

$$\text{mean} = \frac{\text{sum of all values}}{\text{total number of values}}$$

- *Median* is the number in the middle of a set of numbers arranged in numerical order. If there are two numbers in the middle (i.e. an even number in the set) then find the mean of just the two numbers in the middle.

- *Mode* is the number (or numbers) that occurs most frequently in the set. If no number or numbers occur more than once, there is no mode.

Note that all of the above are numerical definitions of "*average*" for a given data set. However, each is computed differently and will often give different results. When the word "average" is utilized within our daily lives it is most often associated with the *mean*. Do not assume that the *mean* is the only *average* of a set of values.

Example 1: Find the mean, median, and mode of the following data sets. Begin by writing the set in increasing order.

a. 5, 1, 4, 5, 3, 1, 5

b. 6 0 6 3 2 2 6 2

YOU TRY

2. Find the mean, median, and mode of the data set 5 2 7 11 6 0 3 3.
Begin by writing the data set in increasing order.

Mean Median Mode

WEIGHTED AVERAGE

A *weighted average* (which is another kind of mean) is used when some values in the number set count more heavily than others. The following examples illustrate this idea.

Example 3: A given Biology class contains 20 students. The 8 female students in the class are enrolled in an average of 14 semester credits. The 12 male students are enrolled in an average of 8 semester credits. Compute the average number of semester credits for the class as a whole. [To begin, circle the GIVENS and underline the GOAL].

Example 4: Grade point average is a classic example of a weighted average. Last term, a student's grades were as indicated in the table below. Compute the student's GPA for the term.

Course	Credits	Grade	Grade Pts	Grade Pt. Totals
Philosophy	**3**	**C**		
English	**3**	**B**		
P.E.	**1**	**A**		
Biology	**5**	**B**		
Total				

YOU TRY

5. Compute the student's GPA for the term.

Course	Credits	Grade
MAT082	**3**	**A**
ENG071	**4**	**B**
PSY 100	**3**	**C**
RDG 061	**3**	**A**

VARIATION, RANGE, & OUTLIERS

Measures of Central Tendency are concerned with finding the most accurate center point or representative point for a given data set. If we want to understand how spread out the data are, then we need to look at the *variation* in the given data.

Example 6: Order the following from least to most variation.

a. The weights of all adults
b. The weights of all adult women
c. The weights of all 20-year-olds
d. The weights of all 20-year-old women

Range is the difference between the largest and smallest value in the set and provides the most information about how spread out the data are. Be sure to write the data set in order before computing the range.

Range = Highest Value – Lowest Value

Example 7: Determine the range of the following data set:

24, 32, 12, 14, 3, 7, 12, 43, 1, 5

YOU TRY

8. Find the range of the following data set 5 2 7 11 6 0 3 3. Start by writing the data set in increasing order.

Example 9: Find the mean, median, mode, and range for the following data sets.

a. 2, 2, 3, 5, 6

b. 2, 2, 3, 5, 20

Outliers are values that are far removed from the other values in a data set. In the above example, data set b has an outlier of 20. Notice how the measures of central tendency and variability are impacted.

TABLES & GRAPHS

Tables and Graphs are often used to display and organize data as illustrated in the examples below. Look for a legend or headers to understand what the different parts of the table or graph represent.

Example 10: A table presents information in rows and columns as shown in this example.

Birth Rates and Populations around the World in 2011

Country	Birth Rate (per 1000 population per year)	Population
French Polynesia	15.53	294,935
Brazil	17.79	203,429, 800
Australia	12.33	21,766,710
Sudan	36.12	45,047,500
Russia	11.05	138,739,900
India	20.97	1,189,173,000
Bulgaria	9.32	7,093,635

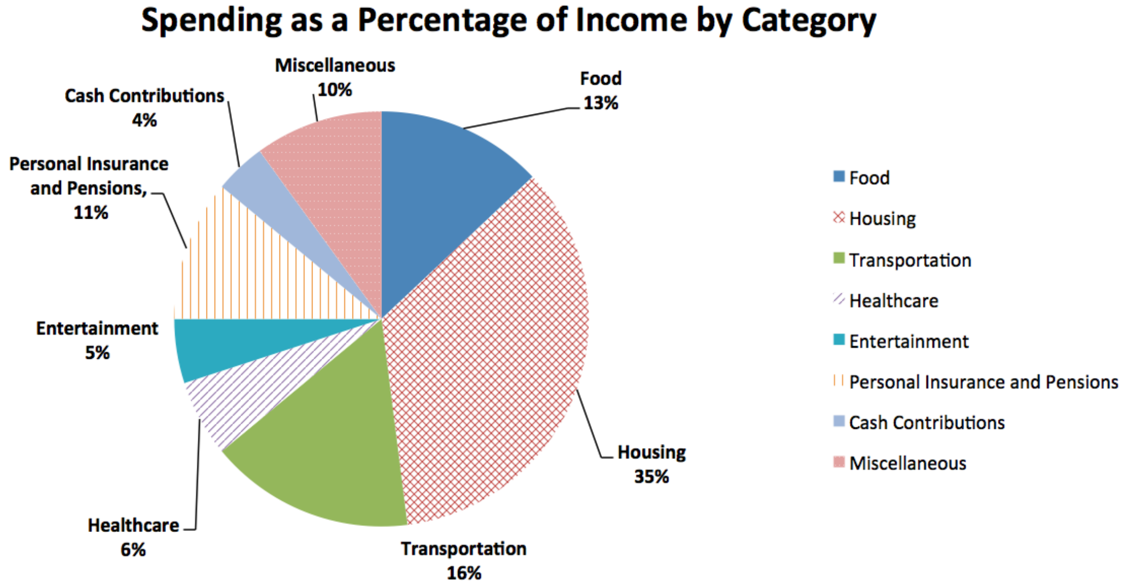
Source: http://www.indexmundi.com/g/

Which country has a birth rate of 17.79? ______________________________

Which country has the smallest birth rate?______________________________

Which country has the largest population? ____________________________

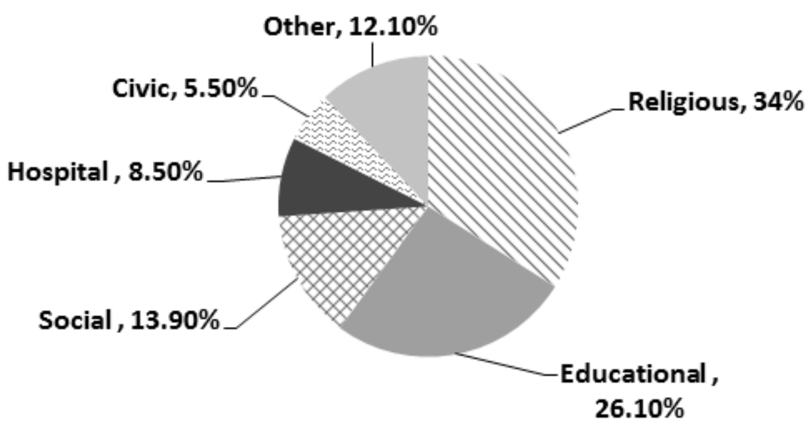
Example 11: A **Circle Graph** (also called a Pie Graph) is used to show how the whole amount is broken up into parts. *(Source: Consumer Expenditure Survey, U.S. Bureau of Labor Statistics, October, 2010)*



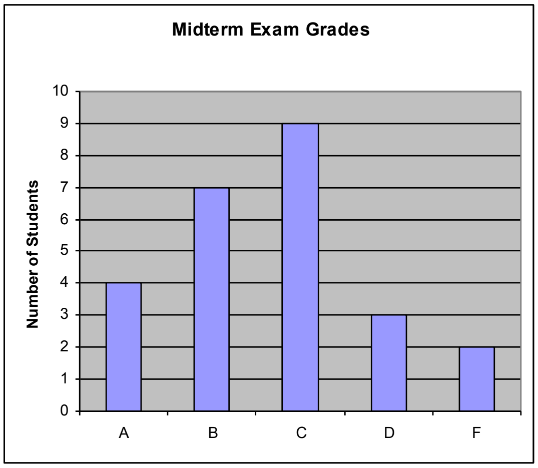
a. How much of their income does the average American spend on healthcare? ________

b. For the average person, what is the single biggest category of expense?_____________

c. Suppose your monthly salary is $2200. How much should you be spending on Food?

YOU TRY
12. In 2009, the Bureau of Labor Statistics reported a surge in volunteerism. At this time, there were a reported 63,361volunteers in the U.S. The pie chart below shows the different categories in which these people volunteered.  Find the number of people who volunteered in an Educational capacity. Round your answer to the nearest whole number.

Example 13: Bar Graph



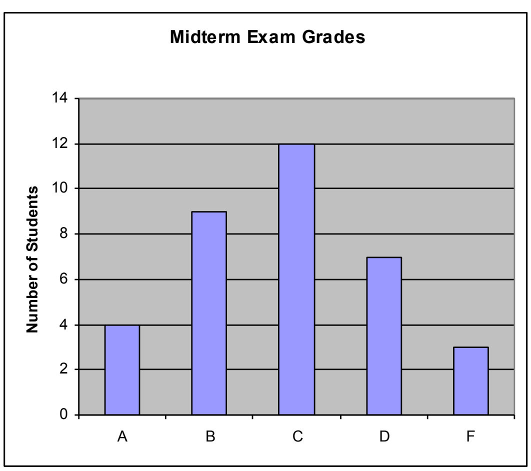
a. How many students made a B on the Midterm?

b. How many students were in the class?

c. What percentage of the class made a B on the midterm? Round to hundredths.

d. What percentage of students made a passing grade (A, B, or C) on the Midterm? Round to hundredths.

You Try



14.

a. How many students made a B on the Midterm?

b. How many students were in the class?

c. What percentage of the class made a B on the midterm? Round to hundredths.

d. What percentage of students made a passing grade (A, B, or C) on the Midterm? Round to hundredths.

LESSON 8 - PRACTICE PROBLEMS

1. Determine the mean, median, mode, and range of the following data sets. Show all of your work. Round to two decimals places as needed.

	Data	Mean	Median	Mode	Range
a.	4, 15, 3, 8, 3, 6, 15, 5, 17				
b.	4, 3, 4, 5, 2, 4, 25				
c.	5, 9,7, 2, 3, 32, 8, 6				
d.	5, 2, 1, 12, 10, 8, 9, 7				
e.	1, 3, 1, 4, 1, 5, 8, 6, 7, 9				

2. Answer True or False for each of the following. If your answer is False, provide an example that proves your point. If your answer is true, explain.

Given a typical set of numerical data with an odd number of values:

a. T or F: The mean is always one of the data values.

b. T or F: The median is always one of the data values.

c. T or F: The mode is always one of the data values.

d. T or F: The range measures the variability of the given data set.

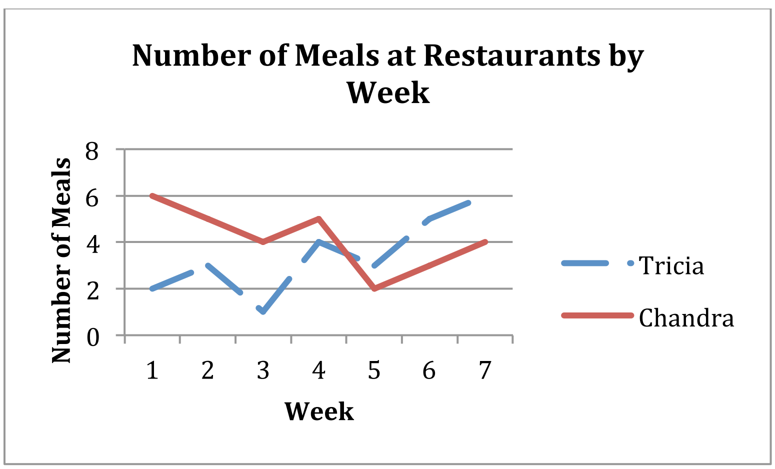
e. T or F: The mean is always the best measure of central tendency to use.

3. Compute the following weighted average. You may need to add information to the given table to help you make the correct computations.

Over a given time period, a convenience store had visits from delivery trucks in the following categories with the indicated charge per delivery. What is the average delivery charge the store pays each week?

Category	Deliveries per week	Per Delivery Charge
Snacks	3	$25.00
Alcohol	2	$100.00
Dairy	4	$75.00

4. Tricia and Chandra love to go to restaurants, but want to save money by eating at home. The double line graph below shows how many meals they ate at restaurants per week for a 7-week time period.



a. How many meals did Tricia eat at restaurants during this 7-week time period?

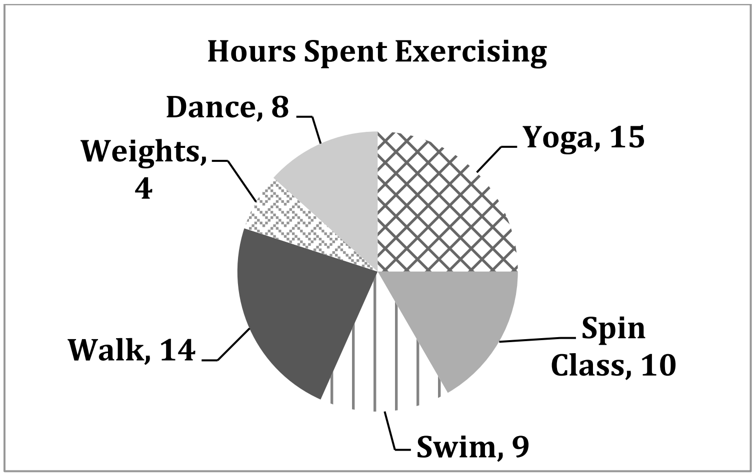
b. What was Tricia's rate of meals at restaurants per week for the given seven-weeks (i.e. compute a unit rate from the given data). Round to two decimals as needed.

c. How many meals did Chandra eat at restaurants during this 7-week time period?

d. What was Chandra's rate of meals at restaurants per week for the given seven-weeks (i.e. compute a unit rate from the given data). Round to two decimals as needed.

e. During which week(s) did Tricia eat more meals at restaurants than Chandra?

5. The graph below displays the number of hours per month that Amber spends in varying exercise activities. Complete each item in the table below including the information in the Total row. DO NOT reduce your fraction answers to lowest terms other than in the Total Fraction of Budget cell. When you are finished, answer the questions below the table.



Category	Amount	Fraction of Exercise	Percent of Exercise
Dance			
Weights			
Walk			
Spin Class			
Swim			
Yoga			
Total			

a. Given the number of hours spent in one month doing Yoga, how many hours would Amber spend on yoga in 12 months?

b. What do you notice about the totals in the Fraction of Budget column and the Percent of Budget column?

c. Reduce all your items in the Fraction of Budget column and then add them together. Do you get the same final, simplified result that you did in the table?

LESSON 8 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Determine the mean, median, mode, and range of the following data sets. Show all of your work. Round to two decimals places as needed.

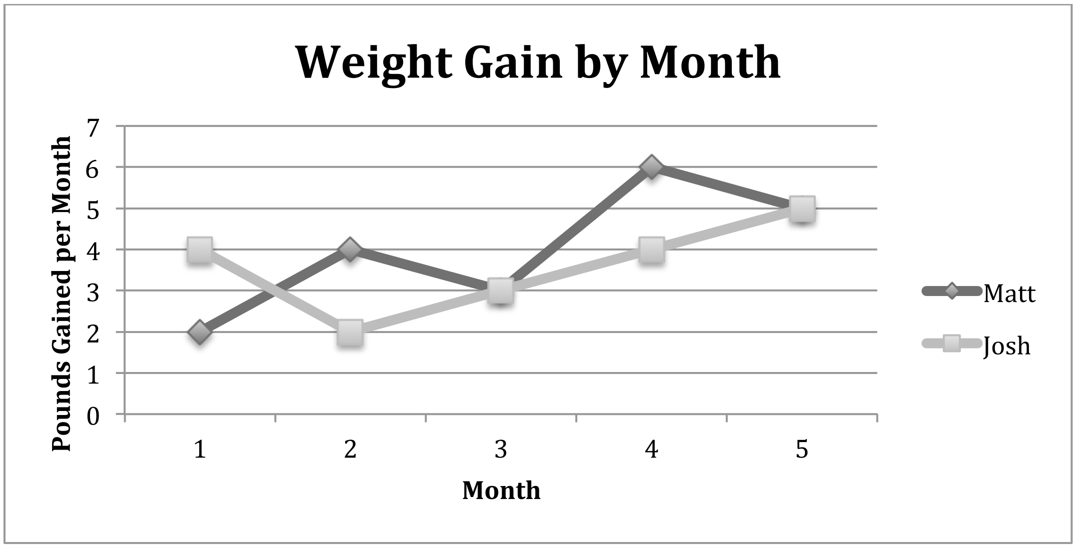
	Data	Mean	Median	Mode	Range
a.	8, 7, 6, 7, 5, 3, 9				
b.	4, 2, 5, 7, 2, 3, 6, 6				

2. Compute the following weighted average. You may need to add information to the given table to help you make the correct computations.

Danielle has started her own exercise company. She charges varying amounts for different classes (charged per month) as shown in the table below. Determine the average charge per person using the weighted average.

Category	Number of People	Dollars per person
Yoga	13	\$17.00
Aerobics	24	\$19.00
Weight Training	7	\$26.00

3. Matt and Josh are wrestlers and begin a diet to gain weight for the season. The line graph below shows the pounds they gained per month over a 5-month time period.



a. How much weight did Matt gain over the 5-month time period?

b. What was Matt's rate of weight gain for the given five months (i.e. compute a unit rate from the given data). Round to two decimals as needed.

c. How much weight did Josh gain over the 5-month time period?

d. What was Josh's rate of weight gain for the given five months (i.e. compute a unit rate from the given data). Round to two decimals as needed.

e. During which month(s) did Josh gain more weight than Matt?

LESSON 9 – UNITS & CONVERSIONS

INTRODUCTION

U.S. units of measure are used every day in many ways. In the United States, when you fill up your car with gallons of gas, drive a certain number of miles to work, or buy a quart of milk you are utilizing *U.S. units of measure*. There is another measuring system also seen in the U.S. but more prevalent in other countries and that is the *metric system*. The *metric system* is easier to use than the U.S. system because all the units are incremented by different powers of 10. The U.S. has never fully converted to the metric system but metric units are all around us as well. In this lesson, we will learn how to work within and between the two systems.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Perform *one-step conversions* of U.S. units	1, 2, YT10
Convert using *U.S. mixed units*	3, 4, 5, 6, YT10
Perform *multi-step conversions* of U.S. units	7, 8, 9, YT10
Add and subtract U.S. units of measure	11, 12, YT13
Perform *conversions* within the metric system	14, 15, YT16
Perform *conversions* between the U.S. and metric system	17, 18, YT19

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- U.S. System of Measure
- U.S. Units and Abbreviations
- U.S. Unit Conversions
- U.S. Mixed Units
- Metric System
- Metric Units and Abbreviations
- Metric Unit Conversions
- U.S./Metric Conversions

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

U.S. UNITS OF MEASURE & SINGLE CONVERSIONS

To understand a little bit about straightforward conversions within the U.S. system, let's look at a typical example:

> Barry is starting a woodworking project. He measures carefully and finds that he needs 8 pieces of wood that are each 18 inches long. He heads to the lumber store and finds that the shortest sections of the wood he needs are sold in lengths of 2 feet. When he buys the 8 pieces he needs, how many inches will he have to remove from each one?

The quickest way for Barry to solve his problem is to do a simple conversion from feet to inches. How many inches are in 2 feet? Here is how the conversion would be performed.

2 feet = ____________ inches	
$\frac{2 \text{ feet}}{1} \cdot \frac{12 \text{ inches}}{1 \text{feet}}$	From the conversion table, 1 ft = 12 in. Set this up as a fraction with feet in the denominator.
$\frac{2 \cdot 12}{1} \cdot \frac{\text{feet}}{\text{feet}} \cdot \frac{\text{inches}}{1}$	Rearrange to write the numbers as a fraction and the units as separate fractions.
24 inches	Remove the like ratios (feet/feet) and simplify the fractions with denominator 1. Multiply to get the final result.
2 feet = 24 inches	

Since boards are sold in lengths of 2 feet = 24 inches, Barry will need to cut 24 – 18 = 6 inches from each board when he gets home.

The conversion steps above are VERY lengthy (and you probably could have done this one in your head) but use as many steps as needed until you get a feel for how these work. In general, for single conversions, the steps below are good to follow.

Recommended Process for single-step US unit conversion problems:

1. Identify the conversion you will need from the table and write it down.
2. Write the conversion as a fraction. Orient that fraction so that the units you no longer want will cancel out and leave the desired units behind.
3. Perform all necessary calculations.

The following table provides a list of the most common U.S. units, abbreviations, and conversions. Reference this table as you work through the rest of the MiniLesson.

US Units/Conversions

Length	Mass/Weight	Area
Units: • Inches (in) • Feet (ft) • Yards (yd) • Miles (mi) Conversions: • 1 ft = 12 in • 1 yd = 3ft • 1 mi = 5280 ft	Units: • Ounces (oz) • Pounds (lb) • Tons Conversions: • 1 lb = 16 oz • 1 ton = 2000 lb	Units: • Square Inches (in^2) • Square Feet (ft^2) • Square Yards ($yd^{2)}$ Conversions: • 144 in^2 = 1 ft^2 • 9 ft^2 = 1 yd^2

Volume	Time
Units: • Ounces (oz) • Cup (c) • Pint (pt) • Quart (qt) • Gallon (gal) • Cubic Feet (ft^3) • Cubic Yard (yd^3) Conversions: • 1c = 8 oz • 1pt = 2c • 1 qt = 2pt • 1qt = 32 oz • 1 gal = 4 qt • 1728 cubic in = 1 cubic ft • 27 cubic ft = 1 cubic yd	Units: • Seconds (sec) • Minutes (min) • Hours (hr) • Days • Weeks (wk) • Months (mo) • Years (yr) Conversions: • 1 min = 60 sec • 1 hr = 60 min • 1 day = 24 hr • 1 wk = 7 day • 1 yr = 52 wk • 1 yr = 12 mo • 1 yr = 365 days

Example 1: Perform each of the following single-step conversions following the recommended process listed on the previous page. Round any decimals to tenths.

a. 4 lb = _____ oz b. 10 yd = _____ ft c. 2.4 pt = _____ c

Example 2: Sarah needs 1.5 cups of ketchup to make her famous meatloaf recipe. She has a brand new, 20-oz bottle of ketchup in her cupboard. How much of this will she need for her meatloaf?

SINGLE UNIT/MULTIPLE UNITS CONVERSIONS

The following examples illustrate additional basic conversions within the U.S. System. A modified form of the conversion process will be used for these problems.

Example 3: Write 26 inches in feet and inches.

Example 4: Write 5 lbs, 6 oz in ounces.

Example 5: Write 30 months in months and years.

Example 6: Write 1 min, 20 sec in seconds.

U.S. MEASUREMENTS & MULTI-STEP CONVERSIONS

Some conversions require more than one step. See how the single-step conversion process is expanded in each of the following problems.

Example 7: How many minutes are in a week?

Example 8: Bryan needs 10 cups of fruit juice to make Sangria. How many quarts of juice should he buy at the grocery store?

Example 9: Raldo measured a room at 9 ft long x 10 ft wide to get an area measurement of 90 square feet (area is length times width). He wants to carpet the room with new carpet, which is measured in square yards. Raldo knows that 1 yd is equivalent to 3 ft so he ordered 30 square yards of carpet. Did he order the correct amount?

YOU TRY

10. Perform each of the following conversions within the U.S. system. Round to tenths as needed. Show complete work.

a. A young girl paced off the length of her room as approximately 8 feet. How many inches would that be?

b. 18 oz = ______ lb

c. 100 yd = ______ ft

d. 10,235 lb = ______ tons

e. How many inches are in 6 feet, 8 inches?

f. How many square inches are in 10 square feet?

ADDING/SUBTRACTING U.S. MEASUREMENTS

Follow the process used in the examples below to add or subtract with U.S. measurements. Circle the GIVENS and underline the GOAL to get started with each problem.

Example 11: Darry recently flew from Phoenix, AZ to Asheville, NC. On his outbound flight, he flew first to Atlanta (3 hours,35 minutes) then to Asheville (45 minutes). What was his total flying time in hours and minutes?

Example 12: Add 1 gal 2 qt and 3 gal 6 qt. Leave your final answer in gallons and quarts.

YOU TRY

13. Rayene needs a board that is exactly 8 inches long to add a little security to the window in her room. She has a board that is 1 ft, 3 inches long to work with. How much, in inches, would she have to cut in order to use the board?

METRIC CONVERSIONS

The *metric system* originated in Europe around 1800 and was quickly adopted around the world as a standard system of measurement. In fact, the U.S. is the only industrialized country that does not use the metric system as its official measurement system even though there are metric units utilized in the U.S. for various things.

The strength of the *metric system* is that it is based on powers of ten as you can see in the chart below. Prefixes are the same for each power of ten above or below the base unit.

Metric Chart						
KILO	HECTO	DEKA		DECI	CENTI	MILLI
1000 x Base	100 x Base	10 x Base	Base Unit	.10 x Base	.01 x Base	.001 x Base
Kilometer (km)	Hectometer (hm)	Dekameter (dam)	Meter (m)	Decimeter (dm)	Centimeter (cm)	Millimeter (mm)
Kiloliter (kl)	Hectoliter (hl)	Dekaliter (dal)	Liter (l)	Deciliter (dl)	Centiliter (cl)	Milliliter (ml)
Kilogram (kg)	Hectogram (hg)	Dekagram (dag)	Gram (g)	Decigram (dg)	Centigram (cg)	Milligram (mg)

Some Common Metric Conversions
1 centimeter (cm) = 10 millimeters (mm)
1 meter (m) = 100 centimeters (cm)
1 kilometer (km) = 1000 meters (m)

The process below works very well for making conversions between metric system units.

Recommended Process for Working Metric Conversion Problems:

1. Use the metric chart above and locate the initial unit and desired unit on the chart (note some charts may be displayed with the smaller units on the left).
2. Count the columns between units (do not count initial unit column).
3. If the desired unit is LARGER than the initial unit, move the decimal to the LEFT the same as the number of columns from step 2. If the desired unit is SMALLER than the initial unit, move the decimal to the RIGHT the same as the number of columns from step 2.

Example 14:

a. 4200 g = ________ mg b. 45 cm = _______ m c. 7,236,137 ml = _______ kl

Example 15: If a person's pupillary distance (from one pupil to the other) is 61 mm and the distance from their pupil to the middle of their upper lip is 7 cm, which distance is longer?

YOU TRY

16. Perform each of the following conversions within the metric system. Show complete work.

a. 1510 m = ________ mm b. 13.50 ml = _______ l c. 5k = ________ m

METRIC/US CONVERSIONS

Although the U.S. relies heavily on our standard measurement system, we do use some metric units. Therefore, we need to know how to move back and forth between the systems.

Some Common Metric/U.S. Conversions			
Length	**Mass/Weight**	**Area**	**Volume**
1 mi = 1.61 km 1 ft = 0.3 m 1 yd = 0.9 m 1 in = 2.54 cm	1 kg = 2.2 lb 1 g = 0.04 oz 1 metric ton = 1.1 ton	1 in^2 = 6.45 cm^2 1 yd^2 = 0.84 m^2 1 mi^2 = 2.59 km^2	I L = 1.1 qt 1 gal = 3.8 L 1 L = 2.1 pt
.621 mi = 1 km 1.094 yd = 1m .394 in = 1cm	.454 kg = 1lb 1 oz = 28.3 g		1 yd^3 = 0.76 m^3 1 in^3 = 16.4 cm^3

Recommended Process for Working Metric/US Conversion Problems:

1. Identify the conversion or conversions you will need from the table and write them down on your paper.
2. Write the conversion as a fraction. Orient that fraction so that the units you no longer want will cancel out and leave the desired units behind.
3. Perform all necessary calculations.

Example 17: Express 5 ml in terms of cups.

Example 18: The country of Cambodia is approximately 700 km from N to S. What would this distance be in miles?

YOU TRY

19. Perform each of the following U.S. to metric or metric to U.S. conversions. Round to hundredths as needed.

a. Soda pop is often sold in 2-liter containers. How many quarts would this be? How many gallons?

b. Your friend Leona is planning to run her first 10km race in a few weeks. How many miles will she run if she completes the race?

c. A roll of Christmas wrapping paper is 3 meters long. How long is this in yards?

d. Convert 10 ml to cups.

LESSON 9 - PRACTICE PROBLEMS

1. Complete each of the following showing as much work as possible.

a. Does it take more cups or gallons to measure the amount of water in a large pot? Explain.

b. The lifespan of a common housefly is about 8 days. How many hours are in 8 full days?

c. A 10k running race is about 6.2 miles. How many feet is this? Assuming that the average person's step is 3 feet long, how many steps are traveled when covering a 10k?

d. Tally the cat is 10.5 pounds. How many ounces is this?

e. Frederika's house gate is 45 inches. How many feet is this? (use decimals)

2. Complete each of the following showing as much work as possible.

a. If you were born on January 1, 1980 at 12:00 am and measured time until January 1, 2013 at 12:00 pm, how many minutes would you have been alive?

b. How many centuries are in 164,240 days? (1 century = 100 years)

c. A container measures 16 inches in length by 2 feet in width by 1 yard in height. If volume is found by multiplying length times width times height, find the volume of the container in cubic feet.

d. Jose's company measures their gains in \$1000's of dollars. If his company earned 6.2 million in gains, how many \$1000's of dollars is this?

e. Tara's pool is 50 yards in length and 20 feet in width. How many square feet is the pool? How many square yards is the pool?

3. Complete each of the following showing as much work as possible.

a. Write 32 months in months and years.

b. 10 years, 6 months is how many months?

c. If your final exam time is 110 minutes, write that time span in hours and minutes.

d. Amy is 14,964 days old today. How old is this is years and days? (Assume 365 days in a year and no leap years). How many days until Amy's birthday?

e. Joseph spent 6.45 hours working on his English paper. How much time is this in hours and minutes?

4. Complete each of the following showing as much work as possible.

a. Add 2 lb 10 oz plus 4 lb 8 oz. Leave your answer in lb, oz.

b. Suppose you took two final exams on a given day. Each final exam allows 110 minutes. You took 1 hour and 5 minutes on the first exam and 50 minutes on the second. How long were you taking exams on that day? How much exam time did you not use on that day?

c. How much greater is 3 gallons than 2 gallons 1 qt?

d. Maria's pool holds 2962.27 gallons of water when filled to the recommended height. She needs to add 57.63 more gallons to reach this height. How many gallons of water are in the pool? How many quarts of water need to be added?

e. Graham ate 9 ounces of protein, 6 ounces of vegetables and 5 ounces of dairy. How many ounces did he eat in total? What is the equivalent weight in pounds?

5. Complete each of the following showing as much work as possible.

a. Which is the best estimate for the capacity of a bottle of olive oil? Choose from 500L, 500ml, 500 g, 500mg and explain your choice.

b. Complete the table. Show your work.

	Centimeters	Meters	Kilometers
Distance from Scottsdale to Las Vegas			421

c. If a tractor-trailer has a mass of 18,245 kg, what is its mass in grams?

d. Which measurement would be closest to the length of a newborn baby? 50 mm, 50 cm, 50 dm, or 50 m?

e. Which measurement would be closest to the weight of a penny? 2.5 mg, 2.5 g, 2.5 kg?

6. Complete each of the following showing as much work as possible.

a. George was riding his bike downhill on a street in a Canadian town. The street-side speed sensor clocked him at 30 km per hour. His bike speedometer was set up in U.S. units of mph. What would the readout have been?

b. A short-course meter pool is 25 meters long. A short-course yard pool is 25 yards long. Which one is longer and by how much (in feet)? Round to two decimal places.

c. In swimming events, a mile in the pool is considered to be 1600 meters. How many meters separate a swimmers mile from an actual mile?

d. At its closest point, the distance from the Moon to the Earth is 225,622 miles. The circumference of the earth is 24,901 miles. How many times would you have to travel around the circumference of the Earth to equal the distance from the Earth to the Moon? (Round to two decimal places)

e. Johanna just returned from a trip to South Africa. She has 7342 rands, the currency of South Africa. She looks up the exchange rate and finds that 1 South African rand = 0.1125 U.S. dollars. What is the value of her money in U.S. dollars?

LESSON 9 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Sher ordered 5 pounds of flour to make cupcakes. How many ounces is this?

2. John biked 3.25 miles to school. How many feet is this?

3. Sheldon spent 9.5 days working on a Physics theorem. How many minutes is this?

4. Jill bought 6.5 yards of fabric. How many inches is this?

5. Bill's daughter is 21 months old. How old is this in years and months?

6. Dan watched 376 minutes of videos for his math class. How long is this in hours and minutes?

7. Sal ate 3 lb. 7 oz. of protein on the first week of his diet and 2 lb. 14 oz. in the second week. How much protein did he eat in the two weeks combined?

8. Jason ran 6.7 km during his morning run. How many meters did he run?

9. The width of a piece of spaghetti is 0.004 m. How many mm is this?

10. In six months, Scott hiked the entire length of the Appalachian Trail from Georgia to Maine. This is a distance of approximately 2200 miles.

a. How many kilometers is this?

b. On average, how many meters did Scott hike per day? (Assume 30 days in a month and round to the nearest meter)

c. If Scott hiked 11 hours per day, what was his average rate in meters per second? Feet per second?

LESSON 10 – GEOMETRY I: PERIMETER & AREA

INTRODUCTION

Geometry is the study of shapes and space. In this lesson, we will focus on shapes and measures of one-dimension and two-dimensions. In the next lesson, we will work with shapes in three-dimensions.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Compute *perimeter* of different shapes	1, 2, YT5
Compute *circumference* of a circle	3, YT5
Explain the difference between *rounded form* and *exact form*	WE4
Compute distance around closed objects of unusual shape	6
Solve applications involving *perimeter* or *circumference*	7
Compute *area* of different shapes	8, 9, 10, YT11
Compute area of nonstandard shapes	12
Solve applications involving *area* and/or *perimeter*	13

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Perimeter
- Closed Triangle
- Geometric Shape/Open Geometric Shape
- Rectangle
- Square
- Circumference
- Circle
- Exact Form
- Rounded Form
- Π
- Diameter
- Radius
- Unit Square
- Area
- Height of a triangle

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

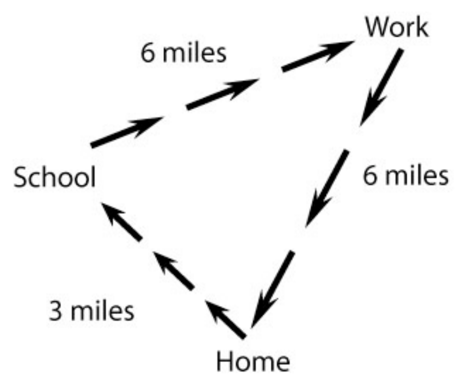
Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

PERIMETER

Perimeter is a one-dimensional measurement that is taken around the outside of a closed geometric shape. Let's start our discussion of the concept of perimeter with an example.

Joseph does not own a car so must ride the bus or walk everywhere he goes. On Mondays, he must get to school, to work, and back home again. His route is pictured below.



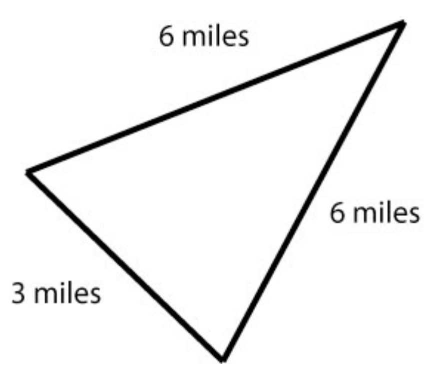
The obvious question to ask in this situation is, "how many miles does Joseph travel on Mondays"?

To compute, we each distance:

$$3 + 6 + 6 = 15$$

Joseph travels 15 miles on Mondays.

Another way to work with this situation is to draw a shape that represents Joseph's travel route and is labeled with the distance from one spot to another.



Notice that the shape made by Joseph's route is that of a closed geometric figure with three sides (a triangle). What we can ask about this shape is, "what is the *perimeter* of the triangle"?

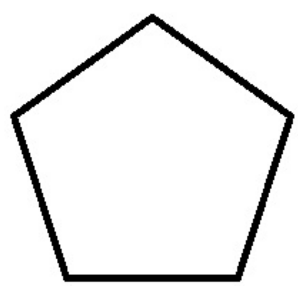
Perimeter means "distance around a closed figure or shape" and to compute we add each length:

$$3 + 6 + 6 = 15$$

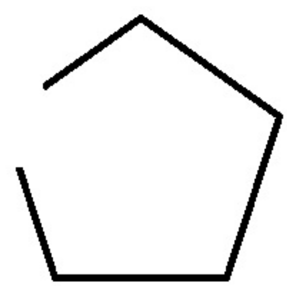
Our conclusion is the same as above. Joseph travels 15 miles on Mondays. However, what we did was model the situation with a geometric shape and then apply a specific geometric concept (*perimeter*) to computer how far Joseph traveled.

Notes on *Perimeter*:
- *Perimeter* is a one-dimensional measurement that represents the distance around a closed geometric figure or shape (no gaps).
- To find *perimeter*, add the lengths of each side of the shape.
- If there are units, include units in your final result. Units will always be of single dimension (i.e. feet, inches, yards, centimeters, etc…)

To compute perimeter, our shapes must be closed. The images below show the difference between a *closed figure* and an *open figure*.



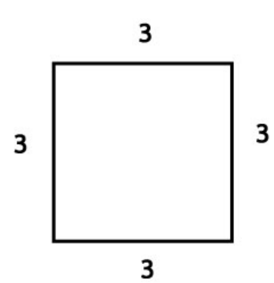
Closed figure:
There is an inside and an outside to the shape. To get from inside to outside, you must cross the boundary of the shape



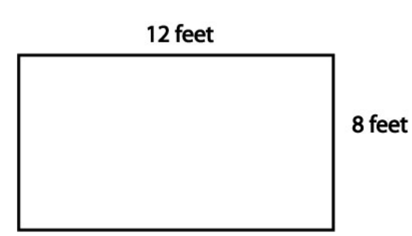
Open figure Not closed:
There isn't an inside or outside. Even a portion that seems enclosed can be reached without crossing the boundary of the shape.

Example 1: Find the perimeter for each of the shapes below.

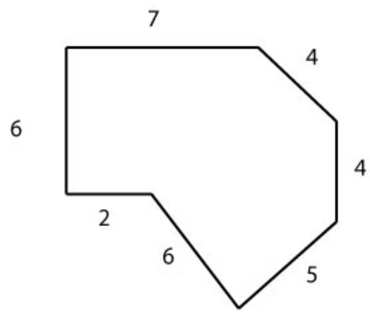
a. Add the lengths of each side.



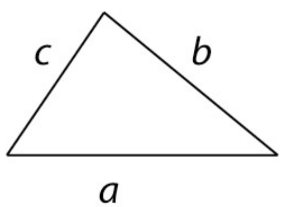
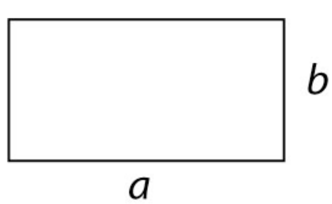
b. Sometimes you have to make assumptions if lengths are not labeled.



Example 2: How do we find the perimeter of this more complicated shape? Just keep adding those side lengths.



If you look closely at the shapes in the previous examples, you might notice some ways to write each perimeter as a more explicit formula. See if the results from what we have done so far match the formulas below.

Shape	Perimeter
Triangle with side lengths *a*, *b*, *c* 	$P = a + b + c$
Square with side length *a* 	$P = a + a + a + a$ $P = 4a$
Rectangle with side lengths *a*, *b* 	$P = a + b + a + b$ $P = a + a + b + b$ $P = 2a + 2b$

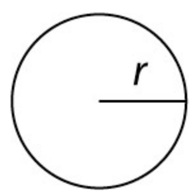
CIRCUMFERENCE

You may realize that we have not yet discussed the distance around a very important geometric shape: a circle! The distance around a circle has a special name called the *circumference*. To find the circumference of a circle, we use the formula below:

$$C = 2\pi r$$

In this formula, π is pronounced "pi", and is defined as the circumference of a circle divided by its diameter, $\pi = \frac{C}{d}$. We usually replace π with the approximation 3.14. The letter *r* represents the *radius* of the circle.

Let's see where the formula for circumference comes from. Below is a generic circle with radius r.



Notes about $C = 2\pi r$

Note: Remember that in the formula, when computing the circumference $C = 2\pi r$, we multiply as follows USUALLY substituting 3.14 in place of π

$$C = 2 \cdot 3.14 \cdot r$$

Often, the use of ( ) will help make the different parts of the formula easier to see:

$$C = (2) \cdot (3.14) \cdot (r)$$

Origins of $C = 2\pi r$

As mentioned earlier, the special number π is defined as the ratio of a circle's circumference to its diameter. We can write this in equation form as:

$$\frac{C}{d} = \pi$$

We know from our previous work that to identify the unknown, C, we can move *d* to the other side of the equation by writing:

$$C = \pi d$$

The diameter is all the way across the circle's middle so the diameter is twice the radius. We can update C in terms of the radius as:

$$C = \pi(2r)$$

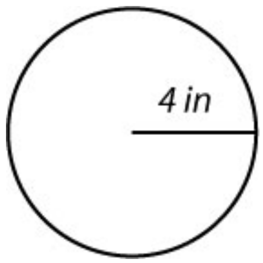
With a little final rearranging of the order our parts are written in, we can say that:

$$C = 2\pi r$$

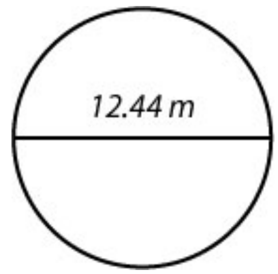
Let's use the formula to find the circumference of a few circles.

Example 3: Find the circumference of each of the following circles. Leave your answers first in *exact form* and then in *rounded form* (to the hundredths place). [Note that when a *radius* is given, its value is centered above a *radius* segment. When a *diameter* is given, its value is centered above a *diameter* segment.]

a.



b.



EXACT FORM VS. ROUNDED FORM

Exact Form vs. Rounded Form

- π is a number in exact form. It is not rounded.
- 3.14 is a rounded form approximation for π

Why does it matter which form we use? It matters because when we round, we introduce error into our final result. For this class, that error is usually acceptable. However, you will find in other subjects such as physics or chemistry, that level of accuracy is a concept of great importance. Let's see an example of the difference in forms.

Worked Example 4: The radius of the moon is about 1079 miles. What is the circumference?

Exact solution	Rounded solution
$C = 2\pi r = 2\pi(1079) = 2158\pi$ To round FROM the exact solution , use the π button on your calculator to get $2158\pi \approx 6779.56$	$C = 2\pi r = 2(3.14)(1079) \approx 6776.12$

Notice that our final results are different. That difference is the error created by using 3.14 as an initial approximation for π. Read the directions carefully on each problem to see which form to use.

YOU TRY

5. Find the circumference or perimeter given in each described situation below. Include a drawing of the shape with the included information. Use the examples to help determine what shapes to draw. Show all work. As in the examples, if units are included then units should be present in your final result. Round to tenths unless indicated otherwise.

a. Find the perimeter of a square with side length 2.17 feet.

b. Find the perimeter of a rectangle with sides of length 4.2 and 3.8.

c. Find the perimeter of a triangle with sides of length 2, 5, 7.

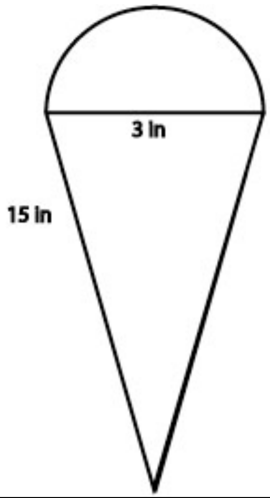
d. Find the circumference of a circle with radius 6 inches. Present answer in exact form and also compute using 3.14 for π. Present rounded form to the nearest tenth.

e. Find the circumference of a circle with diameter 14.8 inches. Present answer in exact form and also compute using 3.14 for π. Present rounded form to the nearest tenth.

FINDING THE DISTANCE AROUND NON-STANDARD SHAPES

The basic formulas for perimeter of straight-line shapes and the circumference of a circle will help us find the distance around more complicated figures as in the example below.

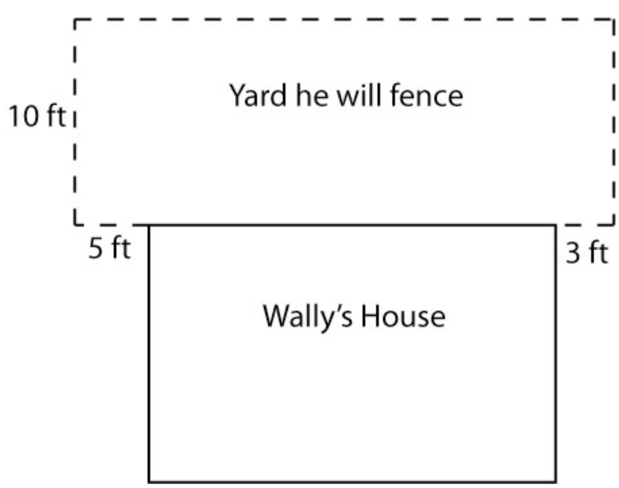
Example 6: Find the distance around the following shape. Round final answer to tenths as use 3.14 for π.



APPLICATIONS OF PERIMETER/CIRCUMFERENCE

Our knowledge of basic geometric shapes can be applied to solve problems like the next example.

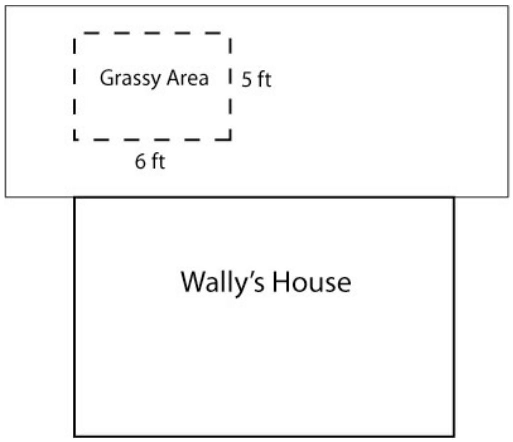
Example 7: Wally wants to add a fence to the back of his house to make some room for his children to play safely (see diagram below). He began measuring his yard but got distracted and forgot to finish measuring before he went to the store. If he remembers that the back wall of his house is 15 yards long, does he have enough information to buy the fencing he needs? If so, how many feet should he buy?



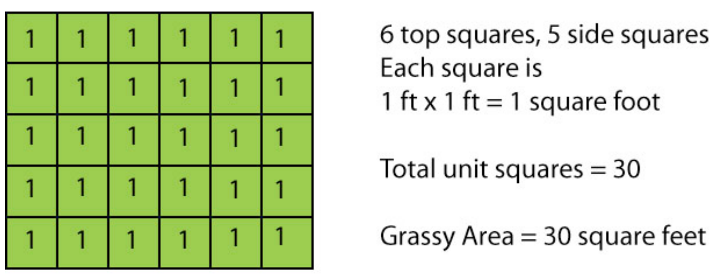
AREA

Let's take another look at Wally's backyard from Example 7 in order to introduce the next concept, *area*.

Wally successfully fenced his yard but now wants to add some landscaping and create a grassy area as shown below.



He heads down to the local lawn store and finds out that in order to determine how much sod he needs, he must figure out the square footage of the area he wants to add grass to. On his way home, he realizes that if he divides the grassy area into sections that are 1 foot by 1 foot and then counts them, he can determine the square footage. Here is the information Wally drew up when he got home.



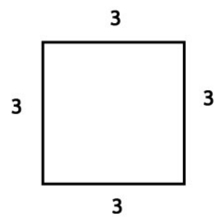
Wally correctly determined the area of the rectangular grassy section to be 30 square feet.

Notes on *Area*:

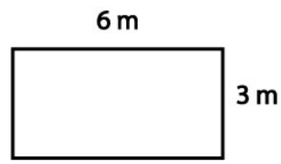
- *Area* is a two-dimensional measurement that represents the amount of space inside a two-dimensional shape.
- To find the *area*, count the number of unit squares inside the shape.
- If there are units, include units in your final result. Units will always be two-dimensional (i.e. square feet, square yards, square miles, etc…)

Example 8: Find the area for each of the shapes below.

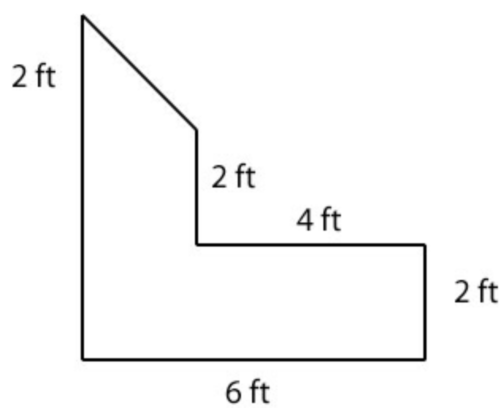
a. Remember to count the unit squares inside the shape.



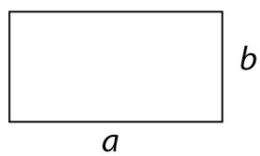
b. Is there a pattern here that would make our work easier?



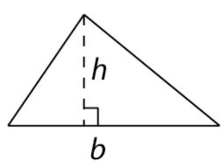
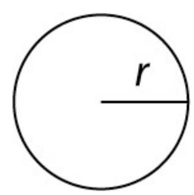
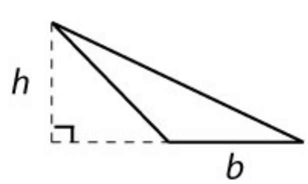
Example 9: How do we find the area for shapes that are more complicated? Break up the areas into shapes that we recognize and add the area values together.



If you look closely at the shapes in the previous examples, you might notice some ways to write each area as a more explicit formula. See if the results from what we have done so far match the formulas below.

Shape	Area
Square with side length a 	$A = a \cdot a$ $A = a^2$
Rectangle with side lengths a, b 	$A = a \cdot b$ *(You will also see this as* *$A = length \cdot width$)*

The area formulas for the shapes below are more complicated to derive so the formulas are listed for you in the table.

Shape	Shape
Triangle with height h and base b  $A = \frac{1}{2}bh = \frac{bh}{2}$ Read as "one-half base times height" Note that h is the straight-line distance from top of the triangle directly to the other side. The small box next to h indicates this. In math terms the box indicates a 90° (right) angle.	Circle with radius r  $A = \pi r^2$ Read as "*pi times radius squared*"
	If your triangle is as pictured at left, then the height is drawn and measured outside the triangle. The area formula is the same.

Example 10: Find the area for each described situation. Create a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for π and round answers to tenths as needed.

a. Find the area of a rectangle whose length is 12.9 meters and height is one-third that amount.

b. Find the area of a triangle with base $24\frac{1}{2}$ inches and height 7 inches.

c. Find the area of a circle with radius $2\frac{1}{3}$ inches. Present answer in exact form and also compute rounded form using 3.14 for π. Present rounded form to the nearest tenth.

YOU TRY

11. Find the area given each described situation. Include a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Round answers to tenths unless otherwise indicated.

a. Find the area of a square with side length 4.2 feet.

b. Find the area of a rectangle with sides of length 4.2 and 3.8.

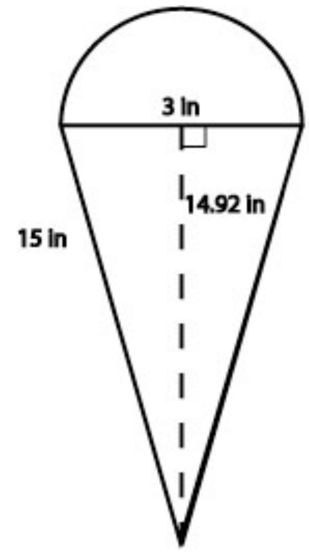
c. Find the area of a triangle with height 7 inches and base 12 inches.

d. Find the area of a circle with radius 6 inches. Present answer in exact form and also compute using 3.14 for π. Present rounded form to the nearest tenth.

FINDING THE AREA OF NON-STANDARD SHAPES

The basic formulas for area will help us find the area of more complicated figures as seen below. This is the same problem we found the perimeter for earlier.

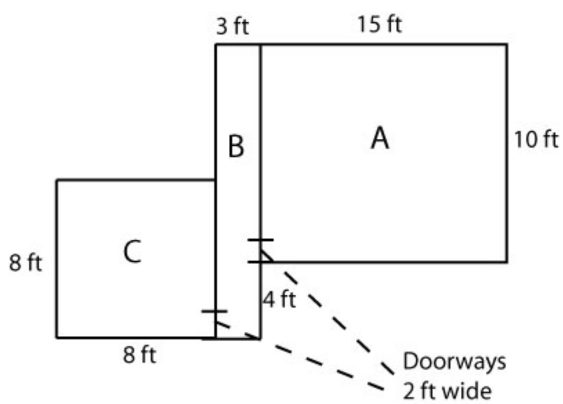
Example 12: Find the area of the given shape. Compute using 3.14 for π and round to the nearest tenth.



APPLICATIONS OF AREA/PERIMETER

We can combine our knowledge of area/perimeter to solve problems such as this one.

Example 13: Wally is still fixing up his house and has a flooring project to complete. He wants to buy enough bamboo flooring to cover the floor space in rooms A, C and hallway B and enough bamboo edging for baseboards in all the spaces as well. How many square feet of flooring and how many feet of baseboards should he buy?



LESSON 10 – PRACTICE PROBLEMS

1. Find the circumference or perimeter given each described situation. Include a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for pi and round answers to tenths as needed.

a. Find the perimeter of a rectangle with height 6 inches and length 12 inches.

b. Find the perimeter of each of the following: a square with side 2 feet, a square with side 4 feet, a square with side 8 feet, a square with side 16 feet.

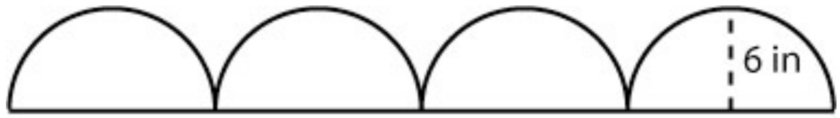
c. Find the circumference of a circle with radius 3 meters.

d. If the circumference of a circle is 324 cm, what is the radius?

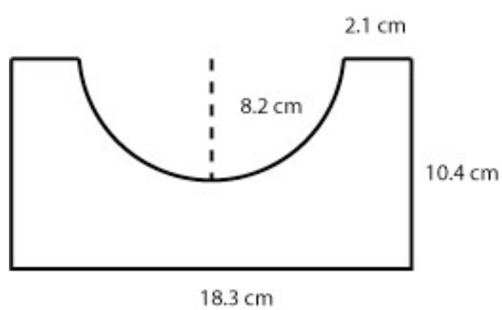
e. Find the perimeter of a triangle with sides of length 6 feet, 5 feet, and 40 inches. Leave your final answer in inches.

2. Find the circumference or perimeter given each described situation. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for pi and round answers to tenths as needed.

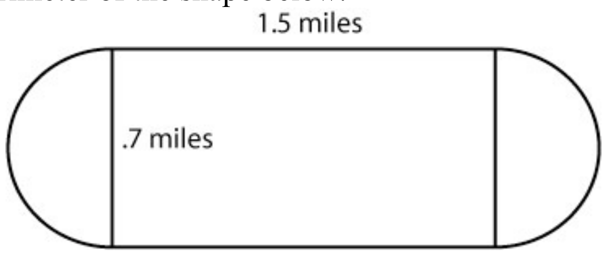
a. If the radius of each half circle is 6 inches, find the perimeter of the object.



b. Find the perimeter of the shape below.



c. Find the perimeter of the shape below.



3. Find the area given each described situation. Include a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for pi and round answers to tenths as needed.

a. Find the area of a rectangle with length 3.45 and width 4.28.

b. Find the area of each of the following: a square with side 2 feet, a square with side 4 feet, a square with side 8 feet, a square with side 16 feet.

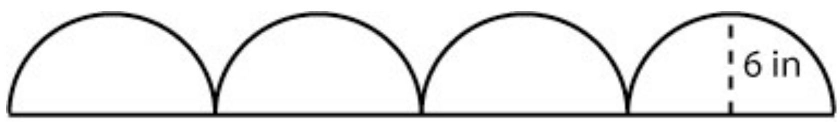
c. Find the area of a triangle with base 4 m and height 12 m.

d. Find the area of a circle with radius 4.56 feet.

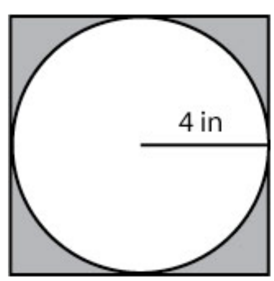
e. Find the area of a rectangle with length 11 m and width 134 cm. Leave your final answer in square meters.

4. Find the area as requested below. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for pi and round answers to tenths as needed.

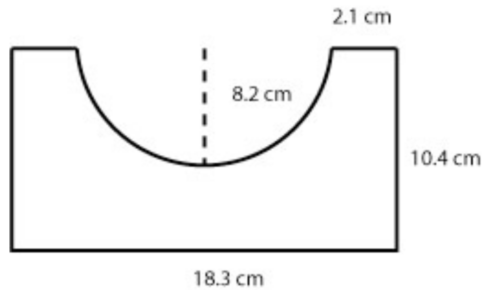
a. If the radius of each half circle is 6 inches, find the area of the object.



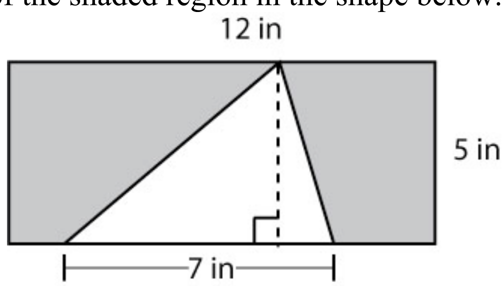
b. Find the area of the shaded region in the shape below.



c. Find the area of the shape below.



d. Find the area of the shaded region in the shape below.



5. Solve the following application problems showing all work. Be sure to circle your GIVENS and underline your GOALS.

a. Draw 4 rectangles each that have area 24 square feet but different perimeters. Try to draw your rectangles with some relative accuracy to each other and include units.

b. In high school, Frank's basketball coach made the team run 15 times around the entire court after every practice. If the boys had to stay outside the lines of the court, what was the least distance they would run? Find the initial distance in feet and then convert to miles. The dimensions of a high school basketball court are 50 feet by 84 feet. If the edges of the court are 2 feet, how much more would someone run that stayed on the inside edge vs. the outside edge? Present your final answer in feet and miles.

c. The radius of the earth is about 3961.3 miles. If a satellite orbits at a distance of 3000 miles above the earth, how many miles would it travel in one trip around the planet?

d. Jarod is painting a room in his house and has a section of wall that will be painted in two colors. The top half of the wall will be white and the bottom half will be lavender. If the wall is 5 meters long and 4 meters high, how much space will he be painting in each color?

e. When the length of a side of a square doubles, how does the area change? Refer to problems 1b and 3b to help you.

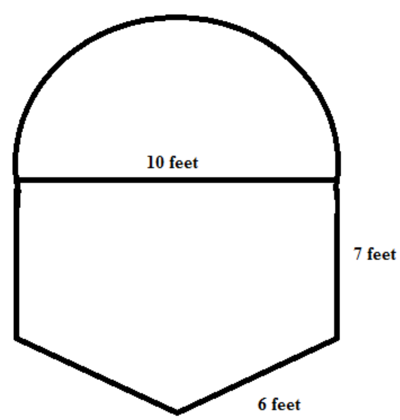
LESSON 10 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Find the perimeter of a rectangle with a length of 22 inches and a width of 3 ft. Include a drawing of the shape with the included information. Write your final answer in inches.

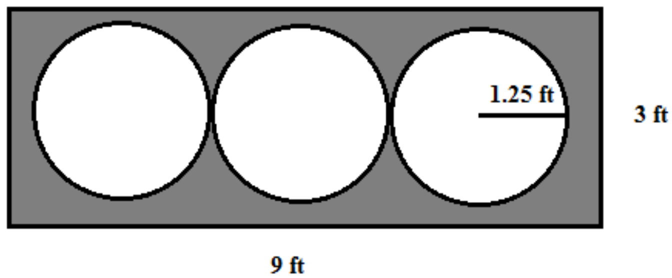
2. Find the circumference of a circle with a diameter of 8 meters. Include a drawing of the shape with the included information. Provide the exact form solution AND a rounded solution using 3.14 for π.

3. Find the perimeter of the object below. Use 3.14 for π and round you final answer to two decimal places.



4. Find the area of a triangle with a base of 2 m and a height of 156 cm. Include a drawing of the shape with the included information. Write your final answer in square meters and round to two decimal places.

5. Find the area of the shaded region in the shape below. Use 3.14 for π and round your final answer to two decimal places.



6. Roberto rides his bike along a rectangular trail that has a length of 7 miles and a width of 1.5 miles. He is thinking of trying a new trail around a circular lake trail that has a radius of 0.9 miles. How many times would he have to ride around the circular lake trail to bike the same distance as his rectangular trail? Use 3.14 for π. Round up to the nearest whole number.

7. Helene has planted a square plot of daisies for her flower business that has a side length of 5 feet. She wants to create a square plot for daffodils that has an area that is 4 times as large. What side length should this square plot have? If she decided to make a rectangular plot for the daffodils instead, what is one way she could do this maintaining the same area?

LESSON 11 – GEOMETRY II: VOLUME & TRIANGLES

INTRODUCTION

We continue our study of *geometry* by working with *volume*, a measure of three dimensions. In addition, we will spend a little more time working with triangles through the concepts of *similar triangles* and also the *Pythagorean theorem*.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Compute the *volume* of different shapes	1, 2, 3, YT4
Solve application problems involving *volume*.	5
Determine the lengths of missing sides in *similar triangles*	6, 7, YT8
Solve application problems involving *similar triangles*.	9
Find *square roots* and determine if roots are *perfect squares*	10, YT11
Use the *Pythagorean theorem* to find the length of a missing side in a *right triangle*.	12, 13, YT14
Solve application problems involving the *Pythagorean theorem*.	15

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Cube
- Unit Cube
- Volume
- Rectangular Solid
- Can/Cylinder
- Sphere
- Angle
- Similar Triangle
- Square Root
- Perfect Square
- Right Triangle
- Hypotenuse
- Pythagorean Theorem

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

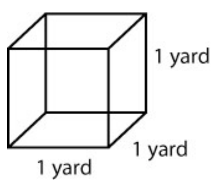
MINILESSON

VOLUME

Let's revisit our friend Wally from Lesson 10 and use another aspect of his yard to introduce the concept of *volume.* Wally is a swimmer and wants to install a lap pool in his backyard. Because he has some extra space, he is going to build a pool that is 25 yards long, 2 yards wide, and 2 yards deep. How many cubic yards of water must be used to fill the pool (assuming right to the top).

Much as we did with *area* (counting unit squares), with *volume* we will be counting unit cubes. What is the volume of a unit cube? Let's look at the shape below:

Finding the Volume of a Cube



Volume = 1 yd x 1yd x yd
= 1 cubic yard

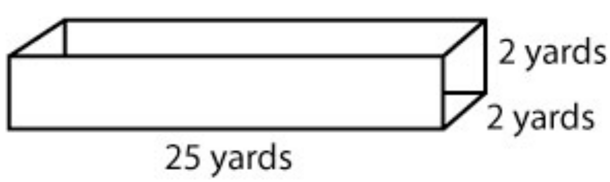
The shape at left is a *cube* (all sides are equal length). In particular, because all sides are of length 1, this cube is called a *unit cube*.

We know the area of the base from our previous work (1 yd x 1 yd or 1 square yard). We are going to take that area and extend it vertically through a height of 1 yard so our volume becomes

Volume = 1 yd x 1 yd x 1 yd = 1 cubic yard.

How does this help Wally? Well, if he can count the number of unit cubes in his pool, he can determine the volume of water needed to fill the pool.

Filling Wally's Pool

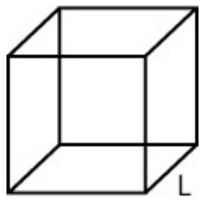
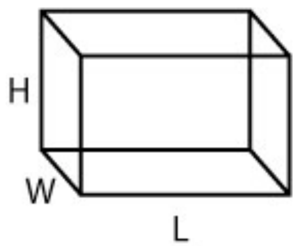


Volume = Length x Width x Height
V = LWH

If we fill the pool with unit cubes, we can fill 25 unit cubes along the length, 2 along the width and 2 along the height. That would give us 25 x 2 x 2 = 100 unit cubes or:

Volume = 25 yd x 2 yd x 2 yd
Volume = 100 cubic yd

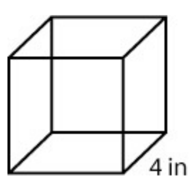
Explicit formulas for the types of *rectangular solids* used on the previous page are:

Shape	Volume
Cube of side length *L* 	$V = L \ x \ L \ x \ L$ $V = L^3$
Box with sides of length *L, W, H* 	$V = L \ x \ W \ x \ H$

Notes on *Volume*:

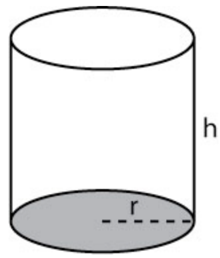
- *Volume* is a three-dimensional measurement that represents the amount of space inside a closed three-dimensional shape.
- To find *volume*, count the number of unit cubes inside a given shape.
- If there are units, include units in your final result. Units will always be three-dimensional (i.e. cubic feet, cubic yards, cubic miles, etc…)

Example 1: Find the volume of each shape below.

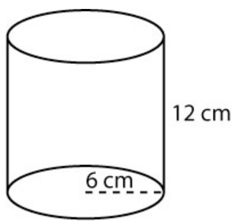
a.	b.
	A box with sides of length 2 ft, 3 ft, $2\frac{1}{2}$ ft.

VOLUME OF A CIRCULAR CYLINDER

Can we use what we know about the area of a circle to formulate the volume of a *can* (also called a *cylinder*)? Take a look at the shape below.

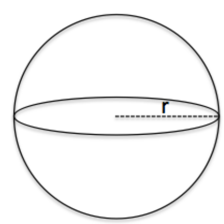
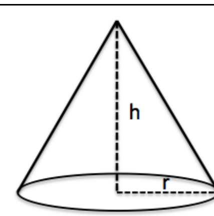
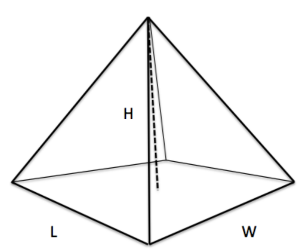
	The base circle is shaded. If we take the area of that circle (A = $A = \pi r^2$) and extend it up through the height *h*, then our volume for the can would be: $V = \pi r^2 h$

Example 2: Find the volume of the cylinder shown below.

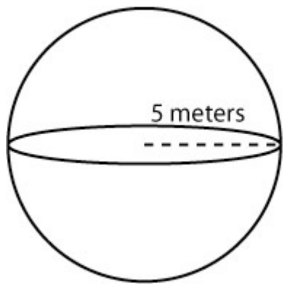


VOLUME OF OTHER SHAPES

The chart below shows the volumes of some other basic geometric shapes.

Shape	Volume	Image
Sphere	$V = \frac{4}{3}\pi r^3$	
Cone	$V = \frac{1}{3}\pi r^2 h$	
Pyramid	$V = \frac{1}{3}LWH$	

Example 3: Find the volume of a sphere with radius 5 meters.



YOU TRY

4. Determine the volume of each of the following. Include a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for π and round answers to tenths as needed.

a. Find the volume of a cube with side 3.25 meters.

b. Find the volume of a box with sides of length 4 feet by $2\frac{1}{2}$ feet by 6 feet.

c. Find the volume of a can with radius 4.62 cm and height 10 cm.

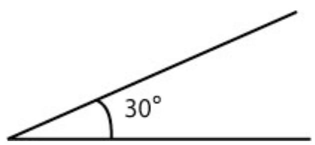
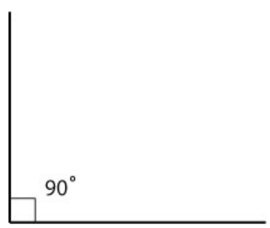
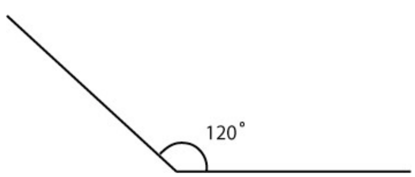
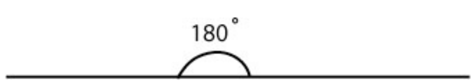
d. Find the volume of a sphere with diameter 12 yards.

APPLICATIONS OF VOLUME

Example 5: If you drank sodas from 5 cans each of diameter 4 inches and height 5 inches, how many cubic inches of soda did you drink? Use 3.14 for π and round to tenths.

ANGLES

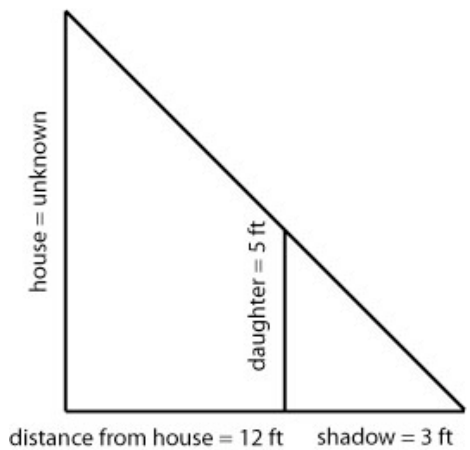
*Angle*s, often measured in *degrees*, measure the amount of rotation or "*arc*" between intersecting line segments. We need to have some sense of what an angle is before moving on to the next topic. See some examples and terminology below.

	• This angle measures 30° • This angle measure is less than 90° • Angles less than 90° are called *Acute Angles*
	• This angle measures 90° • Angles that measure exactly 90° are called *Right Angles* • The small box in the angle corner denotes a right angle
	• This angle measures 120° • This angle measure is more than 90° • Angles that measure more than 90° but less 180° than are called *Obtuse Angles*
	• This angle measures 180° • Angles that measure exactly 180° are called *Straight Angles*

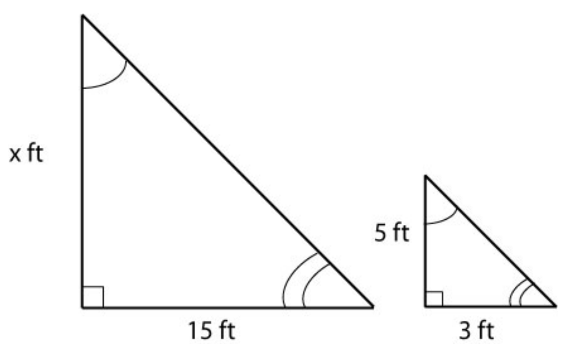
SIMILAR TRIANGLES

Let's begin our discussion of *similar triangles* with an example.

> Mary was out in the yard one day and had her two daughters with her. She was doing some renovations and wanted to know how tall the house was. She noticed a shadow 3 feet long when her daughter was standing 12 feet from the house and used it to set up the drawing below.



We can take that drawing and separate the two triangles as follows allowing us to focus on the numbers and the shapes.



These triangles are what are called *Similar Triangles.* They have the same *angles* and sides in *proportion* to each other. We can use that information to determine the height of the house as seen below.

To determine the height of the house, we set up the following proportion:

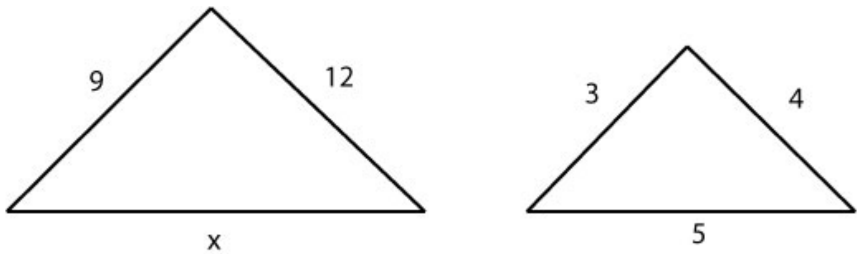
$$\frac{x}{15} = \frac{5}{3}$$

Then, we solve for the unknown x by using cross products as we have done before:

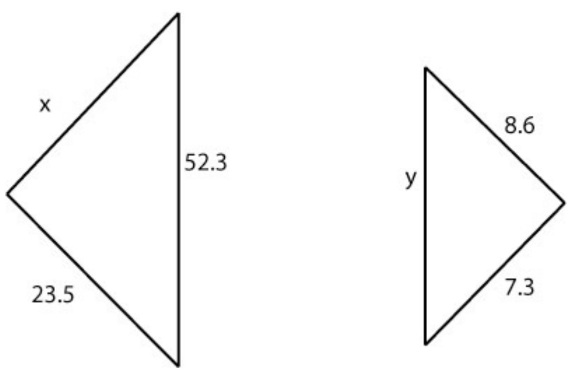
$$x = \frac{5 \cdot 15}{3} = \frac{75}{3} = 25$$

Therefore, we can conclude that the house is 25 feet high.

Example 6: Use the Similar Triangles process to determine the length of the missing side. Set up the proportions in as many ways as possible and show the results are all the same.

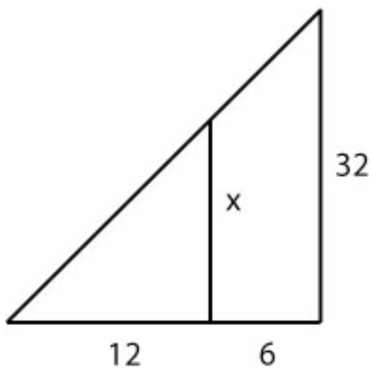


Example 7: Use the Similar Triangles process to determine the length of the missing sides. You may need to redraw your triangles to set up the proportions correctly.



YOU TRY

8. Given the similar triangles below, find the missing lengths. Round to tenths as needed. Feel free to redraw the triangles so you can see the proportional sides.



APPLICATIONS OF SIMILAR TRIANGLES

Example 9: Mary (from the application that started this topic), decides to use what she knows about the height of the roof to measure the height of her second daughter. If her second daughter casts a shadow that is 1.5 feet long when she is 13.5 feet from the house, what is the height of the second daughter? Draw an accurate diagram and use similar triangles to solve.

Before we get to our last topic in this lesson, the *Pythagorean Theorem*, we need to know a little bit about *square roots*.

SQUARE ROOTS

- The *square root* of a number is that number which, when multiplied times itself, gives the original number. On your calculator, look for $\sqrt{}$ to compute square roots.

$$\sqrt{16} = 4 \text{ because } 4 \cdot 4 = 16$$

- A *perfect square* is a number whose square root is a whole number. The square root of a non-perfect square is a decimal value.

$$16 \text{ is a perfect square because } \sqrt{16} = 4$$
$$19 \text{ is NOT a perfect square because } \sqrt{19} \approx 4.36$$

- To obtain a decimal value for non-perfect square roots on your calculator, you may need to change the settings under your MODE button. Check your owner's manual for help if needed.

$$\sqrt{19} \approx 4.36$$

Example 10: Find the square root of each of the following. Round to two decimal places if needed. Indicate those that are perfect squares and explain why.

a. $\sqrt{169}$ b. $\sqrt{31}$ c. $\sqrt{9}$

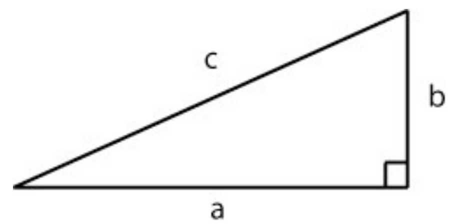
YOU TRY

11. Find the square root of each of the following. Round to two decimal places if needed. Indicate those that are perfect squares and explain why.

a. $\sqrt{225}$ b. $\sqrt{17}$ c. $\sqrt{324}$

THE PYTHAGOREAN THEOREM

The mathematician Pythagoras proved the Pythagorean theorem. The theorem states that given any right triangle with sides a, b, and c as below, the following relationship is always true: $a^2 + b^2 = c^2$

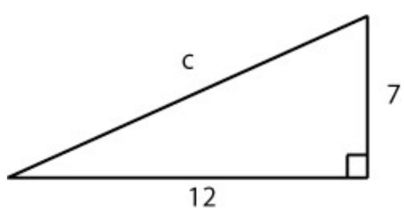


$$a^2 + b^2 = c^2$$

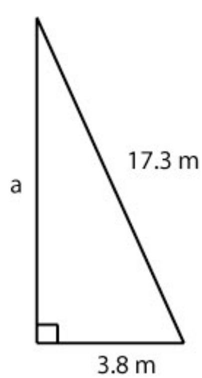
Notes about the Pythagorean theorem:

- The triangle must be a RIGHT triangle (contains a 90° angle).
- The side *c* is called the *Hypotenuse* and ALWAYS sits across from the right angle.
- The lengths *a* and *b* are interchangeable in the theorem but *c* cannot be interchanged with *a* or *b*. In other words, the location of *c* is very important and cannot be changed.

Example 12: Use the Pythagorean theorem to find the missing sides length for the triangle given below. Round to the tenths place.

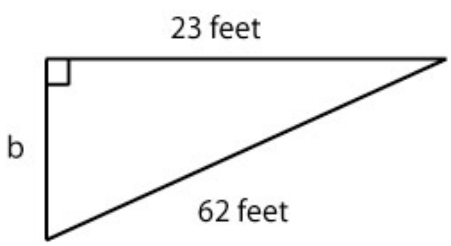


Example 13: Use the Pythagorean theorem to find the missing sides length for the triangle given below. Round to the tenths place.



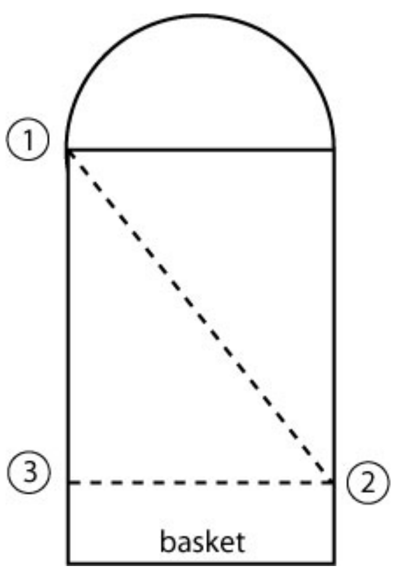
YOU TRY

14. Use the Pythagorean theorem to find the missing sides length for the triangle given below. Round to hundredths.



APPLICATIONS OF THE PYTHAGOREAN THEOREM

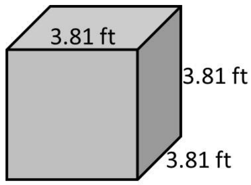
Example 15: In NBA Basketball, the width of the free-throw line is 12 feet (reference: http://www.sportsknowhow.com). A player stands at one exact corner of the free throw line (Player 1) and wants to throw a pass to his open teammate across the lane and close to the basket (Player 2). If his other teammate (Player 3 – heavily guarded) is directly down the lane from him 16 feet, how far is his pass to the open teammate? Fill in the diagram below and use it to help you solve the problem.

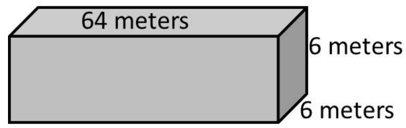


LESSON 11– PRACTICE PROBLEMS

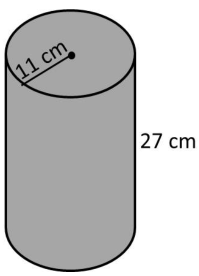
1.

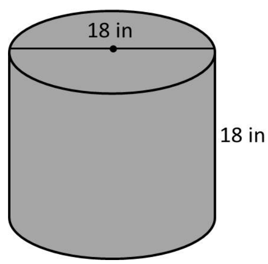
a. Determine the volume of each of the figures shown below. Round your answers to the nearest integer and include appropriate units of measure.

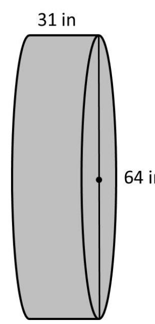




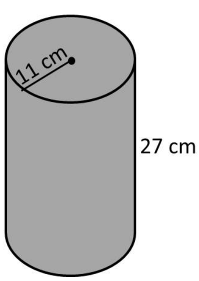
b. Determine the volume of each of the figures shown below. Use 3.14 for π. Round your answers to the nearest hundredth and include appropriate units of measure.

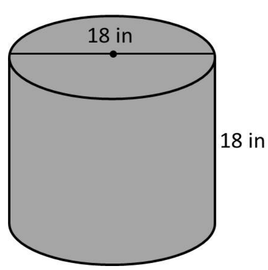


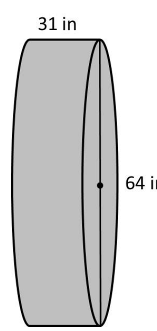




c. Determine the volume of each of the figures shown below. Use the "π" key on your calculator (do not round to 3.14). Round your answers to the nearest integer and include appropriate units of measure.

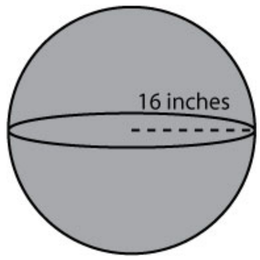


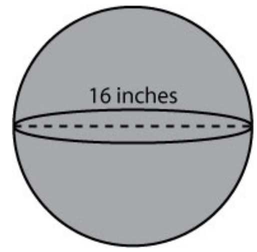




d. Your answers to parts b and c should be different. Why is this the case?

e. Determine the volume of the spheres shown below. Use 3.14 for π. Round your answers to the nearest hundredth and include appropriate units of measure.

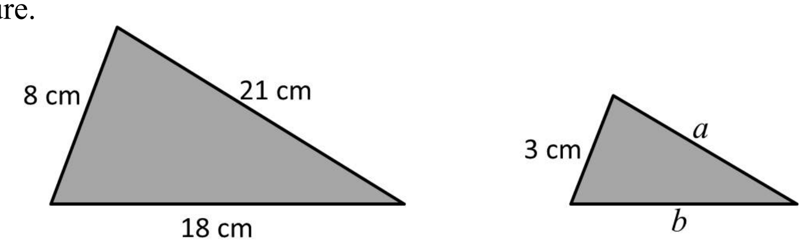




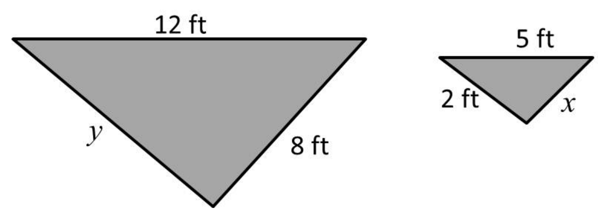
2. Solve the following application problem. Use the 5-step process as your guide. Circle the GIVENS and underline the GOAL. Show MATH WORK and WRITE YOUR FINAL ANSWER AS A COMPLETE SENTENCE. Draw pictures if appropriate.

Renee is interested in buying a hot tub for her backyard and is looking at two models from the same company. Model B is roughly in the shape of a box with dimensions 3 ft x 10 ft x 4 ft. Model A is roughly in the shape of a cylinder with radius 3 ft and height 4 ft. Which one holds a greater volume of water and by how much?

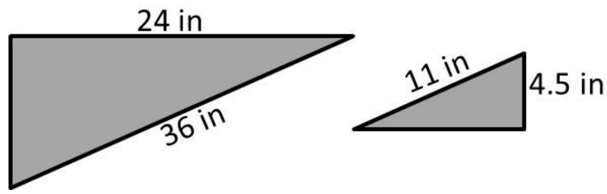
3. Similar Triangles
   a. Use the Similar Triangles process to determine the lengths of the missing sides below. Round your answers to the nearest tenth and include appropriate units of measure.



   b. Use the Similar Triangles process to determine the lengths of the missing sides below. Round your answers to the nearest tenth and include appropriate units of measure.



   c. Use the Similar Triangles process to determine the lengths of the missing sides below. Round your answers to the nearest tenth and include appropriate units of measure.



4. Solve the following application problem. Use the 5-step process as your guide. Circle the GIVENS and underline the GOAL. Show MATH WORK and WRITE YOUR FINAL ANSWER AS A COMPLETE SENTENCE. Draw pictures if appropriate.

Sandy wants so know how tall the flagpole is near her school. One day she decides to find out. She measures the length of the flagpole shadow at 15 feet and measures her shadow at 5 feet (at the same time). Sandy is 4 feet, 8 inches tall. How tall is the flagpole? Give your answer in feet and also in feet and inches.

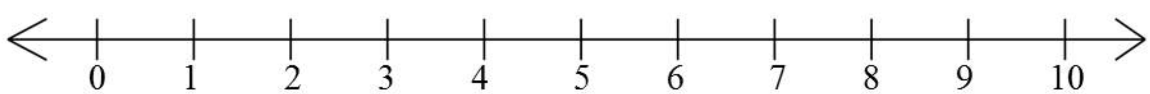
5.

a. Perfect Squares: Without using your calculator, fill in the blanks below.

$\sqrt{1} = \_\_$	$\sqrt{\_\_} = 5$	$\sqrt{\_\_} = 9$
$\sqrt{4} = \_\_$	$\sqrt{\_\_} = 6$	$\sqrt{100} = \_\_$
$\sqrt{9} = \_\_$	$\sqrt{\_\_} = 7$	$\sqrt{\_\_} = 11$
$\sqrt{16} = \_\_$	$\sqrt{\_\_} = 8$	$\sqrt{144} = \_\_$

b. Without using your calculator, place each of the following on the number line below.

$\sqrt{2}$ $\sqrt{11}$ $\sqrt{40}$ $\sqrt{60}$ $\sqrt{99}$

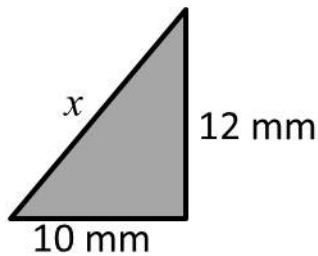


c. Use your calculator to evaluate each of the following. Round your answers to the nearest hundredth.

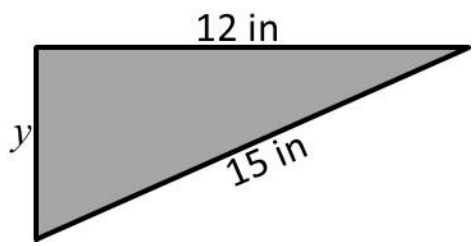
$\sqrt{2} = \_\_\_\_\_$ $\sqrt{11} = \_\_\_\_\_$ $\sqrt{40} = \_\_\_\_\_$ $\sqrt{60} = \_\_\_\_\_$ $\sqrt{99} = \_\_\_\_\_$

6. Use the Pythagorean theorem to find the lengths of the missing sides of the triangles shown below. Round your answers to the nearest tenth and include appropriate units of measure.

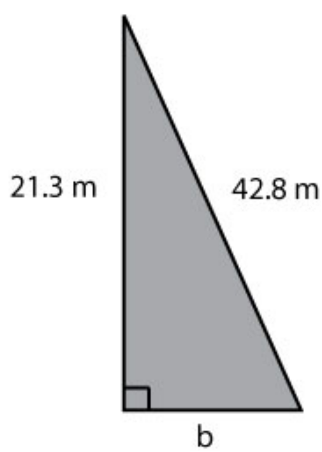
a.



b.



c.



7. Solve the following application problems. Use the 5-step process as your guide. Circle the GIVENS and underline the GOAL. Show MATH WORK and WRITE YOUR FINAL ANSWER AS A COMPLETE SENTENCE. Draw pictures if appropriate.

a. Two trains left a station at exactly the same time. One train traveled south and one train traveled west. When the southbound train had gone 75 miles, the westbound train had gone 125 miles. How far apart were the trains at this time?

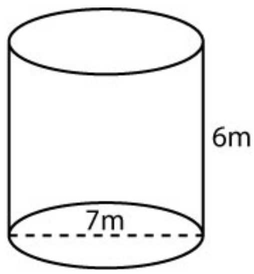
b. TV screens are measured on the diagonal. If we have a TV-cabinet that is 40-inches long and 34 –inches high, how large a TV could we put in the space (leave 2-inches on all sides for the edging of the TV).

LESSON 11 – ASSESS YOUR LEARNING

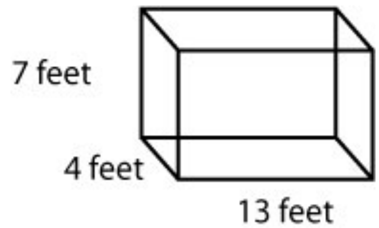
Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Determine the volume of each of the figures shown below. Use 3.14 for π. Round your answers to the nearest hundredth and include appropriate units of measure.

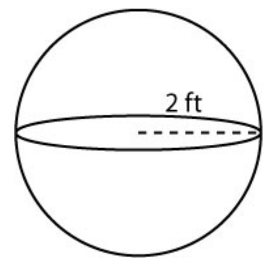
a.



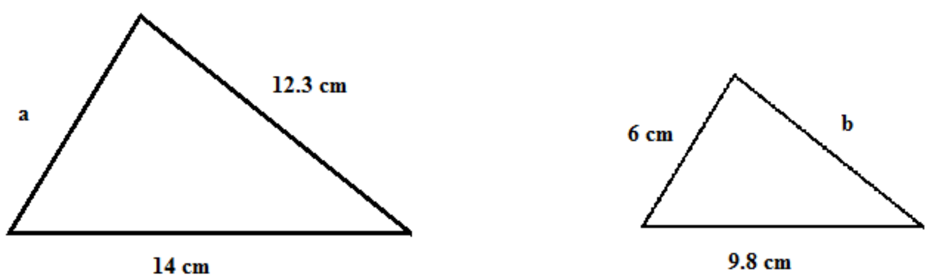
b.



c.



2. Use the Similar Triangles process to determine the lengths of the missing sides below. Round your answers to the nearest tenth and include appropriate units of measure.



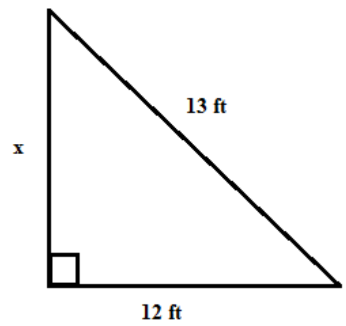
3. Use the chart below to determine what two whole numbers the square roots lie between. Then use your calculator to evaluate each of the following. Round your answers to the nearest hundredth.

$\sqrt{1} = 1$	$\sqrt{25} = 5$	$\sqrt{81} = 9$
$\sqrt{4} = 2$	$\sqrt{36} = 6$	$\sqrt{100} = 10$
$\sqrt{9} = 3$	$\sqrt{49} = 7$	$\sqrt{121} = 11$
$\sqrt{16} = 4$	$\sqrt{64} = 8$	$\sqrt{144} = 12$

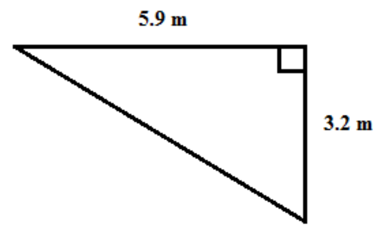
a. $\sqrt{73}$ b. $\sqrt{26}$ c. $\sqrt{52}$ d. $\sqrt{110}$

4. Use the Pythagorean Theorem to find the lengths of the missing sides of the triangles shown below. Round your answers to the nearest tenth and include appropriate units of measure.

4a.



4b.



5. Solve the following application problem. Use the 5-step process as your guide. Circle the GIVENS and underline the GOAL. Show MATH WORK and WRITE YOUR FINAL ANSWER AS A COMPLETE SENTENCE. Draw pictures if appropriate.

a. Mercury is the smallest planet with a radius of only 2,440 km at its equator. Jupiter is the largest of all the planets. It has a radius of 71,492 kilometers at the equator. Maureen makes models of these planets where 1000 km = 1 cm. Find the volume of the models of these planets. Round to the nearest tenth.
Source: http://www.universetoday.com/37120/radius-of-the-planets/#ixzz2EirvutkL

b. Cedric wants to determine the height of his favorite tree. When Cedric's shadow is 8 feet in length, the tree's shadow is 37 feet in length. If Cedric is 5.5 feet tall, how tall is the tree? Write your final answer in feet. Round to two decimal places.

c. Emma's new rectangular smartphone is 12.5 cm in length and 6.5 cm in width. How long is its diagonal? Round to the nearest tenth.

LESSON 12 – SIGNED NUMBERS

INTRODUCTION

In the first few lessons of this course, we learned about different number types including whole numbers and fractions. In this lesson, we will revisit those numbers but with a twist…a twist called "signs". We will add, subtract, multiply, divide, and otherwise combine *signed numbers*. The knowledge gained in this lesson will open a whole new world of applications and number situations and is perhaps the most important foundation for success in an algebra course.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Place *signed numbers* on a number line	1, 2, YT9
Give meaning to *signed numbers*	3, YT7
Work applications that *compare signed numbers*	4, YT10
Compute *absolute value*	5, 6, YT8
Add & subtract *signed numbers*	11, YT12, 13, YT14
Multiply & divide *signed numbers*	15, YT16
Simplify complicated expressions using correct *order of operations*	17, YT18
Solve applications using *signed numbers*	19, YT20

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Signed Numbers
- Integers
- Absolute Value
- Opposite
- Order of Operations
- Three Signs of a Fraction
- Combining Signs

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

SIGNED NUMBERS

If Fred has \$200 in his checking account and he writes a check for \$250, how would we represent his account balance? We could say that:

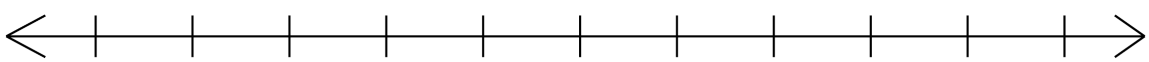
Fred's Balance = \$200 - \$250

But how can we subtract a number that is larger from one that is smaller? Common sense tells us that Fred's account is at a deficit status of \$50. We would say is account balance is -\$50. Therefore,

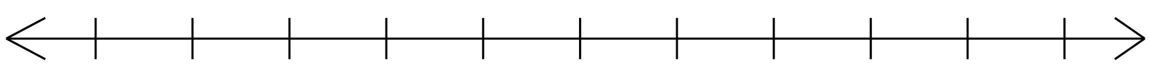
Fred's Balance = \$200 - \$250 = -\$50

In order to deal with situations such as this one, we need to be able to work with both positive numbers and negative numbers. These numbers together are called *signed numbers.*

Example 1: The counting numbers, 0, and the negative counting numbers comprise what are called *integers*. Label the following number line so that it includes 0 and the integers from -5 to 5:



Example 2: Fractions can be signed as well. Label the following number line so that it includes numbers from -1 to 1 in increments of 1/5.



When we *compare signed numbers*, we do so the same way we compare whole numbers. Numbers further to the right on the number line are greater than numbers on the left.

GIVE MEANING TO SIGNED NUMBERS

Similar to the bank example that we started with, these examples illustrate the power of signed numbers to describe a given situation with realistic numbers.

Example 3: Provide a numerical quantity that accurately represents each of the following situations.

a) Tom gambled in Vegas and lost $52.50. ___________________

b) Larry added 25 songs to his playlist. ___________________

c) The airplane descended 500 feet to avoid turbulence. ___________________

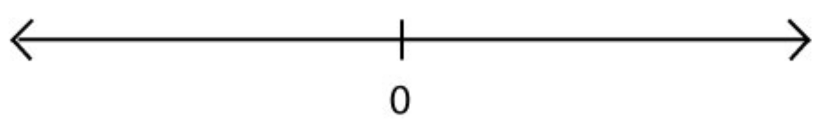
APPLICATIONS WITH SIGNED NUMBERS

Example 4: Camden, SC had a record low temperature of -19°F on Jan 21, 1985, and Monahans, TX had a record low temperature of -23°F on Feb 8, 1933. Which of the two temperatures was the lowest?
(*Data Source Wikipedia: http://en.wikipedia.org/wiki/U.S._state_temperature_extremes*)

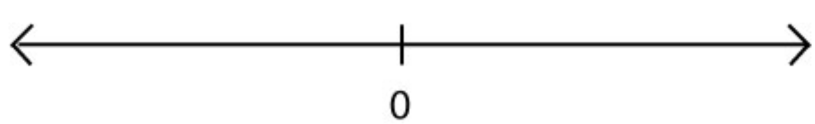
ABSOLUTE VALUE

- The *absolute value* of a number is the distance from the number to 0 on the number line.
- The notation | | is used to represent *absolute value*.
- The *absolute value* of a number is always positive.

Example 5: a. $\left|-\frac{2}{3}\right| =$ b. $|3| =$



Example 6: a. $-\left|\frac{7}{5}\right| =$ b. $-|-1| =$



YOU TRY

7. Determine the signed number that best describes the statements below.

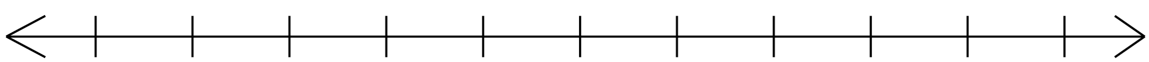
Statement	Signed Number
A balloon dropped 59 feet.	
Lori deleted 324 songs from her iPod.	
A credit card has a balance of $235.34.	
A checking account has a balance of $235.34	

8. a. |-3| = ____________ b. |3| = _____________

c. -|-3| = ____________ d. -|3| = _____________

9. Below the tick marks on the graph, place the numbers -5 to 5, in order from left to right. Place a dot on the graph for each of the numbers in the list and label above the dot with the number (exactly as it appears on the list).

–3, 2.5, |–4|, –1.5, –5

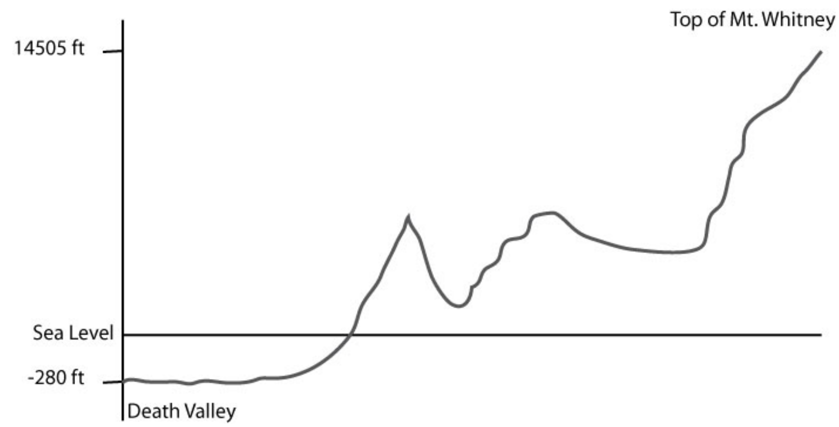


10. Two rides at the amusement park claim to be the greatest thrill. One ride, the Chaotic Coaster drops 100 feet from its highest point. The other ride, the Fearless Flyer, drops 89 feet from its highest point. Which ride drops the most and by how much?

MATHEMATICAL OPERATIONS WITH SIGNED NUMBERS

Now that we know a little bit about signed numbers, let's see how to correctly combine them using mathematical operations or exponents. The following example will get us started:

> When the Badwater Ultra-Marathon in California first began, the race started at Death Valley and ended at the top of Mt. Whitney (see elevation graph below). What was the total elevation gain for the race?



To compute total elevation, we subtract the lower elevation from the higher one.

$$14505-(-280)$$

We can see from the graph that what we really need to do is add 14505 + 280 to get a total elevation of 14,785 feet. How does that work with the symbols we have in the first expression? Subtracting a negative number is the same as adding that number so we can rewrite as follows:

$$14505-(-280)=14505+280=14785$$

The total elevation gain for the race, then, was 14,785 feet.

We will be using PEMDAS as always to help us with correct order of operations.

P	Simplify items inside Parenthesis ( ), brackets [ ] or other grouping symbols first.
E	Simplify items that are raised to powers (Exponents)
M	Perform Multiplication and Division next
D	(as they appear from **Left to Right**)
A	Perform Addition and Subtraction on what is left.
S	(as they appear from **Left to Right)**

In addition to PEMDAS, the following information will help when working with signed numbers.

When working with signed numbers:

- In application problems, use ( ) to separate numbers with negative signs
- Use PEMDAS for order of operations
- Fraction rules and rules for exponents also apply
- Combine signs in a series of simplification steps to simplify your expression
- When combining two signs given together, use the following rules:
  (–)(–) = +, (–)(+) = –, (+)(–) = –, (+)(+) = +
- Use your calculator carefully to help you check results

Example 11: Combine each of the following signed numbers. Use a number line to help you visualize. Show steps if possible. Start by combining signs if possible.

a. $4 + (-3)$ b. $-5 + 8$ c. $-35 + (-20)$

d. $-3.5 - 2.1$ e. $-\frac{1}{4} + (-2)$ f. $-2\frac{3}{4} - (-\frac{1}{4})$

YOU TRY

12. Combine each of the following signed numbers. Use a number line to help you visualize. Show steps if possible. Write improper fractions as mixed numbers.

a. $-3 - (-4)$ b. $\frac{1}{5} - 3$ c. $12 + (-1)$

Example 13: Combine each of the following using correct order of operations. Start by combining the signs if possible.

a. (–20) - 20 + (–10)

b. 8 – (-3) + 4 + (-3) – 2

YOU TRY

14. Combine each of the following using correct order of operations. Start by combining signs if possible.

7 – (-2) + (-1) -5

Example 15: Multiply or divide each of the following. Show steps if possible.

a. $(-8)\cdot(1)$

b. $(-8)\cdot(-1)$

c. $(-2)\cdot(-3)$

d. $(-0.4)^2$

e. $\left(-\frac{1}{3}\right)^2$

f. $(0)\cdot(-3)\cdot(2)$

g. $(-4)\cdot(3)\cdot(-1)$

h. 8 ÷ (-4)

i. $-3\div(-\frac{1}{8})$

THREE SIGNS OF A FRACTION

The following fractions are all equivalent (meaning they have the same value):

$$\frac{-1}{2} = \frac{1}{-2} = -\frac{1}{2}$$

Notice that only the placement of the negative sign is different.

HOWEVER, only the last one, $-\frac{1}{2}$, is considered to be in simplest form.

YOU TRY

16. Multiply or divide each of the following. Show steps if possible.

a. $(-12)\cdot(-1)$

b. $-(-\frac{1}{4})^3$

c. -12 ÷ (-3)

Example 17: Simplify each of the following using correct order of operations and showing all possible steps.

a. $(-10)\left(\frac{1}{5}\right)(8)-3$

b. $\frac{2^3+(-10)}{-4}$

YOU TRY

18. Simplify each of the following using correct order of operations and showing all possible steps. Write improper fractions as mixed numbers.

a. -6 ÷ $(-4)^2$ + 2(-3)

b. $\frac{6-(2-3)^2}{-3+(-5)}$

APPLICATIONS WITH SIGNED NUMBERS

Example 19: On the first five rolls of your FARKLE game in Facebook, you earned 400, 1250, 0, 0, -500 points. What is your total after the 5 rolls? Write your answer in a complete sentence. Start by circling the GIVENS and underlining the GOAL.

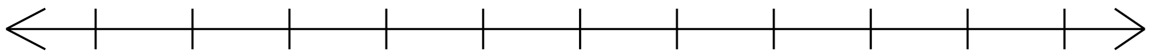
YOU TRY

20. Ryan has an outstanding balance of $2,312.43 on his credit card. If he incurs charges totaling $324.56, makes a payment of $425, and incurs an interest charge of $43.12 what is his new balance? Write your answer in a complete sentence. Start by circling the GIVENS and underlining the GOAL.

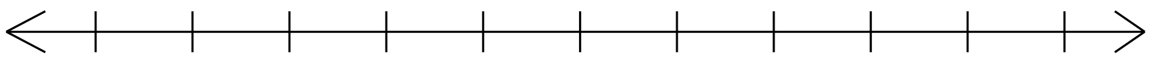
LESSON 12 – PRACTICE PROBLEMS

1. Place numbers on number lines below according to the instructions.

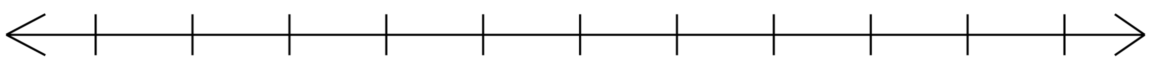
a. Label the number line to include integers from -10 to 0.



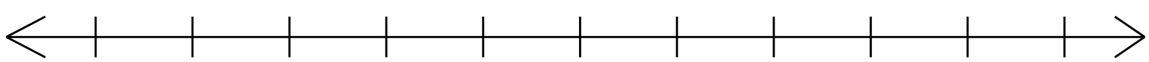
b. Label the number line to include signed numbers from -2½ to 2½ in increments of ½.



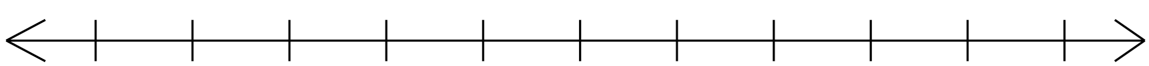
c. Label the number line to include signed numbers from -100 to -90



d. Label the number line to include signed numbers starting with -40, incrementing by ¼.



e. Label the number line to include signed numbers starting with -15, incrementing by 4.



2. Write the signed numbers in order from left to right. Show your work and/or explain your reasoning.

a. -15 5 |-5| $\frac{20}{-5}$ $(-1)^3$

b. -0.5 -0.005 -0.05 -1.5 -1.05

c $\frac{1}{3}$ $\frac{3}{4}$ $\frac{-3}{2}$ $-\frac{4}{3}$ $\frac{1}{-8}$

d. -4 $-\frac{12}{6}$ -6 -2 -3

e. -4.5 -6.8 -7.3 -5.4 -3.8

3. Provide a numerical quantity that accurately represents each of the following situations.

a. In a given week, Mark gained 3 pounds of weight.	
b. Larry descended in a submarine 100 feet below sea level.	
c. The water temperature rose to 100 degrees Fahrenheit.	
d. Ruth spent $54.67 on Amazon.com.	
e. Larry earned $675.23 at his job this week.	

4. Compute each of the following absolute values. Show work if possible.

a. |-5| b. -|-12| c. $-\left|\frac{1}{-4}\right|$

d. $\left|1\frac{2}{3}\right|$ e. -|-2| - |-5|

5. Combine the following signed numbers. Use the number line to help you visualize. Show steps if possible.

a. $\frac{1}{2}-\left(-\frac{1}{2}\right)$ b. $-3+(-5)$

c. $4-(-3)+2$ d. $5+(-1)+4$

e. $6-(-3)+(-1)$

6. Combine the following signed numbers. Show steps if possible. Leave your answer as a simplified fraction. Convert improper fractions to mixed numbers.

a. $\frac{1}{2} \div \left(-\frac{1}{3}\right)$

b. $3 \cdot (-5)$

c. $-4 \cdot (-8)$

d. $-5 \div 4$

e. $2 \div -\frac{1}{3}$

7. Compute each of the following. There are four separate computations per problem.

a. 1^2 $\quad (-1)^2 \quad -(-1)^2 \quad -(1)^2$

b. 2^2 $\quad (-2)^2 \quad -(-2)^2 \quad -(2)^2$

c. $\left(\frac{1}{2}\right)^2 \quad \left(-\frac{1}{2}\right)^2 \quad -\left(-\frac{1}{2}\right)^2 \quad -\left(\frac{1}{2}\right)^2$

d. $\left(\frac{1}{3}\right)^3 \quad \left(-\frac{1}{3}\right)^3 \quad -\left(-\frac{1}{3}\right)^3 \quad -\left(\frac{1}{3}\right)^3$

e. $(1-3)^2 \quad (3-1)^2 \quad -(-3-1)^2 \quad -(1-3)^2$

8. Combine each of the following using correct order of operations and showing all possible steps.

a. $8 \div (-3)^2 + 1(-4)$

b. $3 \cdot (-2)(0) - 4(-5)(0)$

c. $\dfrac{-4-(-2)^2}{7+(-3)}$

d. $-10 \cdot (-4) \div (-8) \cdot 2 + 3$

e. $-(-2)^2 - 1 \div (-5) + 7$

9. Solve each of the following application problems. Use the 5-step process as your guide. Circle the GIVENS and underline the GOAL. Show MATH WORK and WRITE YOUR FINAL ANSWER AS A COMPLETE SENTENCE.

a. According to the yahoo website on Dec 8, 2012 the predicted daily high temperature in Fairbanks, AK for the next 5 days was -10°, -3°, -4°, 8°, 7° Fahrenheit. What would the average daily high temperature be for that 5 day period?

b. An airplane took off and reached a cruising altitude of 34,000 feet. Over the next 4 hours due to weather, the plane descended to 32,000 feet, rose to 35,000 feet, descended to 30,000 feet, and rose to 36,000 ft. Determine the total elevation change during this time.

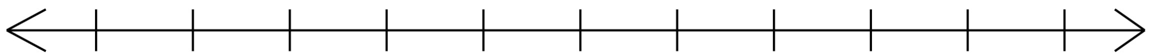
c. Marty has $250.01 in his checking account. He writes checks in the amounts of $13.25, $42.00, $73.45, and $175 and mails them on a Monday. He makes a deposit in the amount of $50.23 on the same day. If all the checks are subtracted from his account on Wednesday, did he overdraw his account and if so, by how much? If not, how much does he have left in his account?

d. Mt. Everest (8,848 meters) is the highest measured point on earth. This point is measured from sea level. Challenger Deep is the lowest measured point on earth at 10,911 meters below sea level. If a person started at Challenger Deep and rose to the height of Mt. Everest, how many vertical meters would they travel? Also compute this distance in miles. Round to hundredths as needed. (*Measurement source*: http://en.wikipedia.org/wiki/Extreme_points_of_Earth)

LESSON 12 – ASSESS YOUR LEARNING

Work the following to assess your learning of the concepts in this lesson. Try to write complete solutions and show as much work as you can. Answers are in the back.

1. Label the number line to include signed numbers starting at -2 in increments of $\frac{1}{3}$.



2. Write the signed numbers in order from left to right. Show your work and/or explain your reasoning.

$$-\frac{3}{5} \qquad -0.65 \qquad -3\frac{1}{3} \qquad -\frac{12}{5} \qquad -3.67$$

3. Provide a numerical quantity that accurately represents each of the following situations.

a. Leonard climbed 8 feet up a ladder.	
b. Tanya scuba dived 15 feet below the surface.	
c. The temperature dropped 18 degrees overnight.	

4. Compute the absolute values.

a. $|7|$ b. $|-7|$ c. $-|-9|$

5. Combine the following signed numbers. Use the number line to help you visualize. Show steps if possible.

a. $7 - 12$ b. $7 - (-4)$ c. $-8 + -4 - \frac{3}{5}$

6. Combine the following signed numbers. Use the number line to help you visualize. Show steps if possible.

a. $-3 \cdot -12$ b. $12 \div \left(-\frac{3}{4}\right)$ c. $6.2 \div -0.31 \cdot -4.7$

7. Compute each of the following.

a. -4^2 b. $(-4)^2$ c. $-(-4)^2$

8. Combine each of the following using correct order of operations and showing all possible steps.

a. $\dfrac{-5+(-2)^2}{-7+(-4)}$ b. $-18 \cdot (-12) \div (-8) \cdot 3 - 7$

9a. The chart below displays the weight loss or gain per week of five friends on a 6-week exercise program. Complete the Total Column and Total Row in the table below. (Note: Since the weight loss or gain is <u>per week</u>, each value in the table is only for that given week not the weeks prior.)

Name	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	**Total**
Bill	+2	+3	0	-2	-1	0	
Frank	-2	-2	-2	-3	-2	-2	
Jillian	-1	-2	0	-1	0	-1	
Sara	-4	-2	-3	-2	-1	-1	
Raj	+2	+1	-1	-1	-1	-1	
Total							

b. In which week(s) was there the greatest weight loss?

c. Which person(s) lost the most weight over the 6 weeks?

d. Determine Raj's average weight change per week over the 6-week period.

e. Find the difference between the greatest weight loss per week total and least weight loss per week total

ANSWERS TO YOU-TRY PROBLEMS

Lesson 1 – Whole Numbers

4: Twelve million, three hundred four thousand, six hundred fifty two
5: 12,000,000
6: 12,304,700
7: Hundred thousand
14: 9
15: 3
17: A one-year membership would cost $633.

Lesson 2 – Introduction to Fractions

4: 1, 2, 3, 6, 8, 18
5: $270 = 2 \times 3^3 \times 5$
7: Answers vary
10 and 12: review video links from lesson media problems
13: $5\frac{2}{11}$
14: $\frac{41}{5}$
18: Some possible answers: $\frac{3}{8} = \frac{6}{16} = \frac{9}{24} = \frac{12}{32} = \frac{15}{40} = \frac{18}{48}$
19: $\frac{5}{9}$
24: $\frac{1}{4}$ of the senators voted against the bill.

Lesson 3 – Fraction Addition & Subtraction

3: a. $\frac{2}{3}$ b. $\frac{6}{13}$

6: a. $\frac{47}{40} = 1\frac{7}{40}$ b. $\frac{1}{15}$

8: a. $\frac{31}{12} = 2\frac{7}{12}$ b. $\frac{81}{8} = 10\frac{1}{8}$

13: a. $\frac{6}{6} = 1$ b. $\frac{21}{3} = 7$

15: Total time was $17\frac{1}{6}$ hr.

17: $\dfrac{107}{120}$

Lesson 4 – Fraction Multiplication & Division

2: a. $\dfrac{40}{5}=8$ b. $\dfrac{35}{8}=4\dfrac{3}{8}$

4: 1175 Hip Hop songs

6: a. $\dfrac{2}{33}$ b. $\dfrac{1}{35}$ c. $\dfrac{13}{2}=6\dfrac{1}{2}$

8: 80 sections

11: a. $\dfrac{27}{343}$ b. $\dfrac{9}{5}=1\dfrac{4}{5}$

14: Bill's wages were $640 the week he worked 56 hours.

Lesson 5 – Decimals

4a: Twelve and six hundred nineteen thousandths

4b: 12.70 4b: The tenths place

6: a. $5\dfrac{3}{8}$ b. $\dfrac{1}{40}$

8: a. 21.240 b. 4.556 c. 3.545

10: 38.38 13: a. $20.05 b. $312

16: 3.05, 3.055, 3.5, 3.55, 3.555, $3\dfrac{3}{5}$

17: Rally had $12.58 left of his initial $40.

Lesson 6 – Percents

3a: $\dfrac{1}{9}=0.1111=11.11\%$

3b: $\dfrac{1}{16}=0.0625=6.25\%$

3c: $\dfrac{4}{5}=0.8000=80\%$

7a: $x=0.15\cdot 324,\quad x=48.60$

7b: $0.2512\cdot 132=x,\quad 33.16=x$

9a: $.40\cdot x=20,\quad x=50$

9b: $105=.1515\cdot x,\quad x=693.07$

11a: $x\cdot 12=8,\quad x=0.67=67\%$

11b: $105=x\cdot 123,\quad x=0.85=85\%$

15: Percent statement – 270 is what percent of 538?

Percent equation $270=x\cdot 538,\ x=.5019=50.19\%$

18a: 62 – 54 = 8. Percent statement 8 is what percent of 54?

Percent equation $8=x\cdot 54,\ x=.1481=14.81\%$ increase

18b: 50 – 40 =10. Percent statement 10 is what percent of 50?

Percent equation $10 = x \cdot 50, \; x = .2000 = 20.00\%$ decrease
21a: 30% of 85 is $25.50 discount. Sale price is $59.50.
21b: I = (14000)(.03)(10) = $4200. Final Balance = $18200.

Lesson 7 – Ratio, Rates & Proportions

5a: $\frac{1}{2}$ [Notice that the units cancelled. This ratio is NOT a rate]
5b: $\frac{4\,\text{geese}}{5\,\text{ducks}}$ [Notice that the units did NOT cancel. This ratio IS a rate.]
9: Option 1 is the better buy per pill.
13a: x = 6
13b: p = 8.33
15: Mary will earn $140.63 in 15 hours this week.

Lesson 8 – Statistics

2: Mean 4.625, Median 4, Mode 3
5: The student's GPA for the term was 3.23.
8: The range = 11.
12: 16537 people volunteered in an Educational capacity.
14a: 9
14b: 35
14c: 25.71%
14d: 71.43%

Lesson 9 –Units & Conversions
10a: 96 in
10b: 1.1 lb
10c: 300 ft
10d: 5.1 tons
10e: 80 in
10f: 1440 square inches or 1440 in^2
13: 7 inches
16a: 1,510,000 mm
16b: 0.01350 l
16c: 5000 m
19a: 2.20 qt, 0.53 gal
19b: 6.21 mi
19c: 3.28 yd
19d: 0.04 c

Lesson 10 – Geometry: Perimeter & Area
5a: 8.7 feet
5b: 16
5c: 14
5d: Exact 12π in, Rounded 37.7 in
5e: Exact 14.8π in, Rounded 46.5 in
11a: 17.6 ft^2 or 17.6 square feet
11b: 16.0
11c: 42.0 in^2 or 42.0 square inches
11d: Exact 36π in^2 or 36π square inches, Rounded 113.0 in^2 or 113.0 square inches

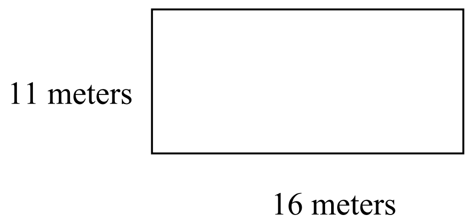
Lesson 11 – Geometry: Volume
4a: 34.4 m^3 or 34.4 cubic meters
4b: 60.0 ft^3 or 60.0 cubic feet
4c: 670.2 cm^3 or 670.2 cubic centimeters
4d: 904.3 yd^3 or 904.3 cubic yards
8:x = 21.3
11a: 15 (perfect square)
11b: 4.12 (not a perfect square)
11c: 18 (perfect square)
14: 57.58 feet

Lesson 12 – Signed Numbers
7: -59, -324, 253.34, 253.34
8a: 3
8b: 3
8c: -3
8d: -3
9: Numbers should be placed as indicated by the directions in the following order:
-5, -3, -1.5, 2.5, |-4|
10: Chaotic Coaster drops the most by 11 feet.
12a: 1
12b: $-\frac{14}{5} = -2\frac{4}{5}$
12c: 11
14: 3
16a: 12
16b: $\frac{1}{64}$
16c: 4
18a: $-\frac{51}{8} = -6\frac{3}{8}$
18b: $-\frac{5}{8}$
20: The new balance is \$2255.11.

BASIC ARITHMETIC - CUMULATIVE REVIEW

For video solutions to these problems, visit: http://bit.ly/ArithmeticReview

1. Find the perimeter and area of the rectangle shown below:

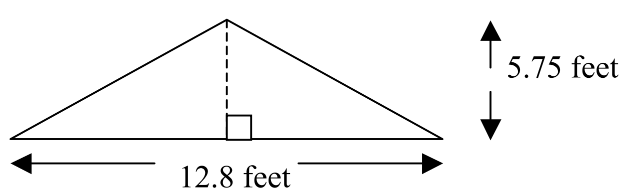


2. Find the circumference and area of a circle with radius 5 feet. Use $\pi = 3.14$.

3. A rectangle is 3.35 inches long and 7.3 inches wide. Find its area and perimeter.

4. If a circle has a diameter of 5.37 centimeters, what is the length of its radius?

5. Find the area of the triangle shown below:



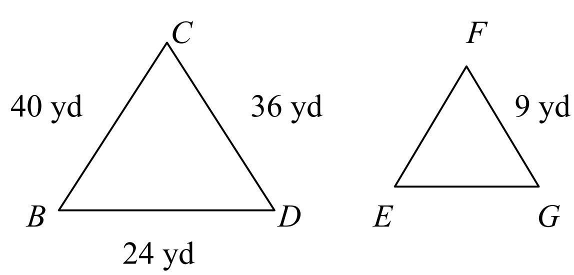
6. Classify the following angles as *acute, right, obtuse* or *straight*. Then make a sketch of each angle.

   a. 65° b. 113° c. 180° d. 90°

7. The hypotenuse of a right triangle has a length of 15 inches and one of the triangle's legs has a length of 8 inches.
   a. What is the length of the other leg of the triangle? Round your answer to 3 decimal places.

   b. Make a drawing of the triangle labeling each side with its length.

8. Triangle *BCD* is similar to triangle *EFG*. Find *EF* and *EG*.



9. Add 3 ft. 7 in. and 8 ft. 9 in.

10. Convert 96 ounces to pounds.

11. Multiply 9036 by 309.

12. Add 0.68 + 299 + 5.2 + 0.385.

13. Subtract 3.542 from 5.82.

14. Divide 414 by 6.

15. Multiply 2.002 by 0.048.

16. Divide 9 by 0.06.

17. Multiply: $\frac{4}{33} \times \frac{3}{8}$

18. Divide: $6 \div 2\frac{1}{6}$

19. Add; write answer in simplest form: $\frac{26}{20}+\frac{36}{20}+\frac{12}{20}$

20. Add: $\frac{2}{5}+\frac{1}{15}+\frac{1}{6}$

21. Subtract: $\frac{11}{12}-\frac{1}{8}$

22. Simplify: $\frac{2}{5}\div\frac{1}{2}+\frac{5}{6}$

23. Write the ratio of 48 to 70 as a fraction in simplest form.

24. Find the unknown number in the following proportion:

$$\frac{3}{4}=\frac{n}{52}$$

25. Write 2.3 as a percent.

26. Write $\frac{5}{16}$ as a percent.

27. Find 20% of $213.58. Round to the nearest cent.

28. Convert 2.3 meters to centimeters.

29. Convert 5 square yards to square feet.

30. Convert 2356 millimeters to meters.

31. Use the formula $V = \frac{4}{3}\pi r^3$ to find the volume of a sphere with a radius of 3 inches. Use 3.14 for π.

32. Find the volume of a cube with side of 4 centimeters.

33. Simplify: $36 \div 6 \times 2 - 4 + 17$

34. Simplify: $20 - (6 + 16) + 2^2$

35. Simplify: $45 + 60 \div 15 - 3.5(-2)$

36. Subtract; write answer in simplest form: $\frac{1}{2} - \frac{1}{6}$

37. Compute $3^2 + (-5)^2$.

38. Round 36,924,563 to the nearest ten thousand.

39. Round 124.36887 to the nearest thousandth.

40. Round $182.279 to the nearest cent.

41. When Marion was getting freight ready for shipment she made a row with 10 identical boxes that was 870 centimeters long. How long was each box?

42. There are 20 people on our swim team. One fourth of the team went to a swim meet in April. How many people went to the swim meet in April?

43. Peter walked a distance of 2 miles to deliver a storage box. He stopped every $\frac{1}{3}$ mile to rest. How many times did Peter stop?

44. Sam drank $\frac{3}{5}$ of a quart of milk. Harry drank $\frac{5}{8}$ of a quart. How much more did Harry drink than Sam?

45. Linda made a triple batch of sugar cookies. She used $5\frac{1}{8}$ cups of flour. Before she made her cookies she had $8\frac{2}{3}$ cups of flour. How much flour does Linda have left?

46. Jane's monthly gross pay is \$3014.74. If she has the following deductions, what is her net pay?

Federal Tax: \$450.69	Savings Plan: \$24.00	FICA: \$244.38
State Tax: \$112.57	Insurance: \$233.16	

47. Kelly's car used 10.36 gallons of gas to go 317.33 miles. Estimate the number of miles per gallon Kelly's car gets by rounding your answer to the nearest hundredth.

48. A bus travels 90 miles on 6 gallons of gas. How many gallons will it need to travel 165 miles?

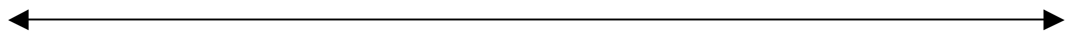
49. Lewis works at a nursery. Last fall he kept track of bulb sales and discovered that $\frac{7}{10}$ of the bulbs sold were variegated tulip bulbs. Write this fraction as a percent.

50. Katie sold 195 chocolate bars; 40% had coconut. How many chocolate bars had coconut?

51. A coat regularly selling for $46.85 is advertised at 25% off. Find the sale price to the nearest cent.

52. The sales tax rate in a certain state is 6%. Find the total price paid for a pair of shoes that costs $48.

53. Locate the following numbers on the number line: $-2.5, 0, \frac{1}{2}, 5.5 \text{ and } -4.$



54. Evaluate: $|-8+5|$

55. The temperature was -7° F at 6:00 am one day in Detroit. A cold front lowered the temperature over the next hour by 2° F. What was the temperature at 7:00am?

56. Arrange the following from smallest to largest: $\frac{3}{5}, \frac{10}{20}, \frac{5}{8}, \frac{85}{100}$

57. Arrange the following from smallest to largest: 0.073, 0.7, 0.07, 0.072, 0.0731

58. Compute 17% of 455.

BASIC ARITHMETIC - CUMULATIVE REVIEW ANSWERS

1. P = 54 meters A = 176 square meters

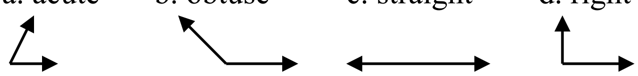
2. C = 31.4 feet A = 78.5 square feet

3. P = 21.3 inches A = 24.455 square inches

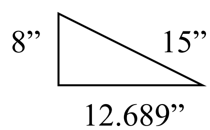
4. radius = 2.685 cm

5. A = 36.8 square feet

6. a. acute b. obtuse c. straight d. right



7. a. 12.689 inches b.



8. EF = 10 yards EG = 6 yards

9. 12 feet 4 inches

10. 6 pounds

11. 2,792,124

12. 305.265

13. 2.278

14. 69

15. 0.096096

16. 150

17. $\frac{1}{22}$

18. $\frac{36}{13}$

19. $3\frac{7}{10}$ *or* $\frac{37}{10}$

20. $\frac{19}{30}$

21. $\frac{19}{24}$

22. $1\frac{19}{30}$ *or* $\frac{49}{30}$

23. $\frac{24}{35}$

24. $n = 39$

25. 230%

26. 31.25%

27. $42.72

28. 230 cm

29. 45 square feet

30. 2.356 meters

31. 113.04 cubic inches

32. 64 cubic cm

33. 25

34. 2

35. 56

36. $\frac{1}{3}$

37. 34

38. 36,920,000

39. 124.369

40. $182.28

41. 87 cm

42. 5 people

43. 6 times

44. $\frac{1}{40}$ quart

45. $3\frac{13}{24}$ cups

46. $1949.94

47. 30.63 miles per gallon

48. 11 gallons

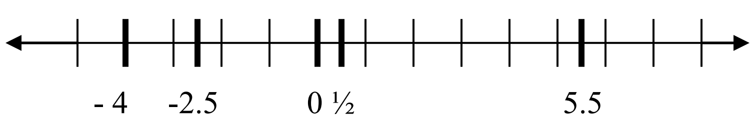
49. 70% were tulip bulbs

50. 78 had coconut

51. $35.14

52. $50.88

53.



54. 3

55. - 9° F

56. $\frac{10}{20}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{85}{100}$

57. 0.07, 0.072, 0.073, 0.0731, 0.7

58. 77.35